Math 110 Common Exam III
November 30, 2016

Time: 1 hour and 25 minutes

Instructions: Show all work for full credit.
No outside materials or calculators allowed.

Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: ____________________________

ID #: ____________________________

Instructor/Section: ____________________________

"I pledge by my honor that I will abide by the NJIT Academic Integrity Code."

____________________________ (Signature)

Relevant Formulas for this Exam

Given \( \triangle ABC \) as shown to the right:

\[
\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}
\]

\[
a^2 = b^2 + c^2 - 2bc \cos(A)
\]

\[
b^2 = a^2 + c^2 - 2ac \cos(B)
\]

\[
c^2 = b^2 + a^2 - 2ab \cos(C)
\]

Area of \( \triangle ABC \) = \( s(s - a)(s - b)(s - c) \), where \( s = \frac{a+b+c}{2} \)

Show all your work. Simplify and reduce all answers as much as possible. Rationalize all denominators.
1. a) (5 pts) Solve the following system of two linear equations:
\[
\begin{align*}
3x + 2y &= 19 \\
3(x - 4y) &= -3
\end{align*}
\]

- Elimination Method

\[
\begin{align*}
-3x + 12y &= 9 \\
14y &= 28 \\
y &= 2
\end{align*}
\]

* Plug 2 in for y

\[
\begin{align*}
3x + 4 &= 19 \\
x &= 5
\end{align*}
\]

(5, 2)

b) (10 pts) Solve the following system of linear equations by Substitution or Elimination. Note: If you are using the Substitution Method, start with the first equation and isolate for x in terms of y and z, then substitute for x in the other two equations.
\[
\begin{align*}
x - y + z &= 2 \\
2x + y - 2z &= -2 \\
3x - 2y + z &= 2
\end{align*}
\]

* Elimination Method

\[
\begin{align*}
5 - 2(x - y + z = 2) &\Rightarrow -2x + 3y - 2z = -4 \\
+ ax + y - 3z = -2 \\
3y - 4z &= -6 \\
-3(x - y + z = 2) &\Rightarrow -3x + 3y - 3z = -6 \\
3x - 2y + z &= 2 \\
y - 2z &= -4
\end{align*}
\]

System of 2 Equations, 2 unknowns

\[
\begin{align*}
3y - 4z &= -6 \\
-3(y - 2z = -4) &\Rightarrow -3y + 6z = 12 \\
2z &= 6 \\
z &= 3
\end{align*}
\]

Now solve for x:

\[
\begin{align*}
x - 2 + 3 &= 2 \\
x + 1 &= 2 \\
x &= 1
\end{align*}
\]

(1, 2, 3)
2. (10 pts) Given the triangle $\triangle ABC$ with $\angle A = 150^\circ, b = 3$ and $c = \sqrt{3}$, solve for side ‘a’ and find the area of the triangle.

$\star$ 2 sides & included angle: Law of Cosines

\[
\begin{align*}
a^2 & = (b)^2 + (c)^2 - 2(bc) \cos \angle A \\
& = 9 + 3 - 2(3)(\sqrt{3}) \cos 150^\circ \\
& = 12 - 6\sqrt{3} \left(\frac{-\sqrt{3}}{2}\right) \\
& = 12 + 9 \\
a^2 & = 21 \\
a & = \pm\sqrt{21} \\
\boxed{a = \sqrt{21}}
\end{align*}
\]

\star Need 2 sides and included angle

Find Area $\quad K = \frac{1}{2} ab \sin C$

\[
\begin{align*}
\frac{1}{2} (3)(\sqrt{3}) \sin 150^\circ \\
\frac{1}{2} (3)(\sqrt{3})(\frac{1}{2}) \\
K = \frac{3\sqrt{3}}{4} \text{ sq. units}
\end{align*}
\]
3. (10 pts) In a triangle $\triangle EFG$, $e = 10$, $\angle E = 45^\circ$ and $\angle F = 60^\circ$. Solve the triangle for $\angle G$ and side $f$.

\[ \angle G = 180^\circ - (60 + 45) \]
\[ \angle G = 75^\circ \]

Find $f$: \textbf{Law of Sines}

\[ \frac{f}{\sin 60} = \frac{10}{\sin 45} \]

\[ f = \frac{\sin 60(10)}{\sin 45} \]

\[ f = \frac{10 \left( \frac{\sqrt{3}}{2} \right)}{\frac{\sqrt{2}}{2}} \]

\[ f = \frac{5\sqrt{3} \cdot \sqrt{2}}{1} \]

\[ f = \frac{5\sqrt{6}}{1} \]

\[ f = 5\sqrt{6} \]
4. a) (6 pts) Solve the following trigonometric equation for all possible solutions of x in the interval $[0, 2\pi)$.
\[ 2 \sin x \cos(2x) - \sin x = 0 \]

\[ \sin x \left[ 3 \cos(2x) - 1 \right] = 0 \]
\[ \sin x = 0 \]
\[ x = 0 \]
\[ x = \pi \]
\[ \cos 2x = \frac{1}{2} \]
\[ 2x = \frac{\pi}{3} + 2n\pi \]
\[ x = \frac{\pi}{6} + n\pi \]
\[ n = 0: \quad x = \frac{\pi}{6} \]
\[ n = 1: \quad x = \frac{5\pi}{6} \]

b) (5 pts) Solve the following trigonometric equation for all possible solutions of x. Express the solutions in radians.
\[ 2 \sin(x) + \cos^2(x) + \sin^2(x) = 0 \]

\[ 2 \sin(x) + 1 = 0 \]
\[ \sin x = -\frac{1}{2} \]
\[ x = \frac{7\pi}{6} \pm 2n\pi \]
\[ x = \frac{11\pi}{6} \pm 2n\pi \]

\[ \cos^2 x + \sin^8 x = 1 \]

\[ \frac{1 - x}{x} = \sqrt{5} \quad \text{(rationalize your answer)} \]

\[ 1 - x = \sqrt{5} x \]
\[ 1 = x + \sqrt{5} x \]
\[ 1 = x(1 + \sqrt{5}) \]
\[ x = \frac{1}{1 + \sqrt{5}} \cdot \frac{1 - \sqrt{5}}{1 - \sqrt{5}} = \frac{1 - \sqrt{5}}{1 - 5} = \frac{1 - \sqrt{5}}{-4} = \frac{\sqrt{5} - 1}{4} \]
5. (10 pts) Suppose a triangle $\triangle ABC$ has side lengths of $a = 4$, $b = 5$ and $c = 8$, find the area of the triangle.

Using Heron's Formula:

$$s = \frac{4 + 5 + 8}{2}$$

$$s = \frac{17}{2}$$

$$s - a = \frac{17}{2} - 4 = \frac{9}{2}$$

$$s - b = \frac{17}{2} - \frac{10}{2} = \frac{7}{2}$$

$$s - c = \frac{17}{2} - \frac{16}{2} = \frac{1}{2}$$

Area = $\sqrt{\frac{17}{2} \left( \frac{9}{2} \right) \left( \frac{7}{2} \right) \left( \frac{1}{2} \right)}$

$$\sqrt{\frac{17(9)(7)(1)}{16}}$$

$$\frac{1}{4} \sqrt{17(9)(7)}$$

$$\frac{3}{4} \sqrt{7(17)}$$

$$\frac{3}{4} \sqrt{119} \text{ units}$$
6. (15 pts) Graph the following equation on the axis given below. Show all intercepts, center points, vertices and end points, as appropriate.

a) \((x - 2)^2 + (y + 1)^2 = 25\)

Center: \((2, -1)\)

\(r = 5\)

Circle

b) \((x - 4) + 2(y + 3) = 4\)

\(x - 4 + 2y + 6 = 4\)

\(x + 2y = 2\)

\(\begin{array}{c|c}
\text{x} & \text{y} \\
\hline
0 & 1 \\
2 & 0
\end{array}\)

Line

c) \(\frac{(x+1)^2}{25} + \frac{(y-3)^2}{9} = 1\)

\(C = (-1, 3)\)

Major Axis parallel to x-axis

Minor Axis parallel to y-axis

Ellipse

\(a^2 = 25\)

\(b^2 = 9\)

\(a = 5\)

\(b = 3\)

\(c = \sqrt{25 - 9} = \sqrt{16}\)

\(c = \pm 4\)

Foci: \(f_1 = (3, 3)\)

\(f_2 = (-1, 3)\)
7. a) (5 pts) Consider the rectangular curve: \( y^2 = 4x \). Convert this equation into a polar curve of the form \( r = f(\theta) \). Note: \( r = 0 \) is a trivial curve and can be discarded. Simplify \( r = f(\theta) \) to a form that is a product of two of the six basic trigonometric functions.

\[
\begin{align*}
y^2 &= 4x \\
(r \sin \theta)^2 &= 4r \cos \theta \\
\frac{r^2 \sin^2 \theta}{r} &= 4r \cos \theta \\
r \sin^2 \theta &= 4 \cos \theta \\
r &= \frac{4 \cos \theta}{\sin^2 \theta} \\
&= 4 \cdot \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta} \\
&= 4 \cot \theta \csc \theta
\end{align*}
\]

b) (5 pts) Write the equation in standard form: \( 9x^2 + 4y^2 - 18x + 16y - 11 = 0 \). Identify its type.

\[
\begin{align*}
9x^2 - 18x + 4y^2 + 16y - 11 &= 0 \\
9(x^2 - 2x + 1) - 1 + 4[(y^2 + 4y + 4) - 4] - 11 &= 0 \\
9(x-1)^2 - 9 + 4(y+2)^2 - 16 - 11 &= 0 \\
\frac{9(x-1)^2}{36} + \frac{4(y+2)^2}{36} &= \frac{36}{36} \\
\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} &= 1
\end{align*}
\]

Ellipse
8. a) (10 pts) Sketch the graph of the polar curve: \( r = 2 + 2 \sin \theta \) on either of the axes below.

\[
\begin{align*}
    r(0) &= 2 + 0 = 2 \\
    r\left(\frac{\pi}{2}\right) &= 2 + 2 = 4 \\
    r(\pi) &= 2 + 0 = 2 \\
    r\left(\frac{3\pi}{2}\right) &= 2 + (-2) = 0 \\
    r(2\pi) &= 2 + 0 = 2
\end{align*}
\]

b) (5 pts) Find the intersection point(s) in polar coordinates \((r, \theta)\) where the curve \( r = -2 \sin \theta \) intersects the curve \( r = 2 + 2\sin \theta \).

\[
\begin{align*}
    2 + 2 \sin \theta &= -2 \sin \theta \\
    2 + 2 \sin \theta + 2 \sin \theta &= 0 \\
    3 \sin \theta &= -2 \\
    \sin \theta &= -\frac{2}{3}
\end{align*}
\]

\[
\begin{align*}
    \theta = &\frac{\pi}{6}, & \frac{11\pi}{6} \\
    \left(1, \frac{7\pi}{6}\right), & \left(1, \frac{11\pi}{6}\right)
\end{align*}
\]

c) (4 pts) Graph the curve: \( r = -2 \sin \theta \) on the same set of axes as the curve above, labeling the point(s) of intersection found above in part b.

\[
\begin{align*}
    r &= -2 \sin \theta \\
    r &= -2 \sin \left(\frac{\pi}{6}\right) \\
    &= -2 \left(-\frac{1}{2}\right) \\
    &= 1
\end{align*}
\]