1. (a) (4 points) Find the point of intersection of the line \( x = 2t, y = t - 1, z = 4 - t \) and the plane \( 2x + y + 2z = 10 \).
(b) (10 points) Find the equation of the line of intersection of the planes
\[
x + y - z = 5 \quad \text{and} \quad 3x + y + z = 3.
\]

2. (a) (7 points) A surface has equation \( x^2 + 2xy - y^2 + z^2 = 8 \). Find the equation of the tangent plane and the equation of the normal line to the surface at the point \( P(2, -1, 3) \).
(b) (7 points) Find the directional derivative of \( f(x, y) = x^4 - x^2y^3 \) at \( P(1, 1) \) in the direction of \( u = 4i + 3j \). Also, find the unit vector in the direction in which \( f(x, y) \) increases most rapidly and find the rate of increase in that direction.

3. (a) (7 points) Find all local maxima, minima, and saddle points of \( f(x, y) = x^3 + y^3 - 3xy + 4 \), and evaluate \( f(x, y) \) at each point.
(b) (7 points) Use the method of Lagrange multipliers to find the maximum and minimum values of \( x^2 + y^2 \) subject to the constraint that \( x^2 - 2x + y^2 - 6y = 0 \).

4. (a) (7 points) Sketch the region of integration, reverse the order of integration, and evaluate the integral
\[
\int_0^1 \int_{3y}^3 e^{x^2} \, dx \, dy.
\]
(b) (7 points) A region is bounded above by the paraboloid \( z = 9 - x^2 - y^2 \), below by the plane \( z = 0 \), and lies outside the cylinder \( x^2 + y^2 = 1 \). Sketch the region. Use cylindrical polar coordinates \( (r, \theta, z) \) to set up a triple integral that gives the volume of the region, then evaluate the integral to find the volume.

5. (a) (7 points) Evaluate the line integral \( \int_C (y - x) \, dx \) along the path \( C: x = t, y = 2t + 1 \) for \( 0 \leq t \leq 3 \).
(b) (7 points) Evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) for the vector field \( \mathbf{F} = 2yi + 3xj + (x+y)k \) along the curve \( C: \mathbf{r}(t) = (\cos t)i + (\sin t)j + (t/6)k \) from the point \( A \), where \( t = 0 \), to the point \( B \), where \( t = 2\pi \).

6. Given the vector field
\[
\mathbf{F} = 3x^2i + \frac{z^2}{y}j + (2z\ln y + z)k
\]
(a) (4 points) Show that the vector field \( \mathbf{F} \) is conservative, or equivalently, show that it satisfies the conditions of the component test, or curl test \( \nabla \times \mathbf{F} = \mathbf{0} \).
(b) (8 points) Find a scalar potential \( f(x, y, z) \) for the vector field \( \mathbf{F} \). Show all steps of your construction of the potential \( f(x, y, z) \).
(c) (2 points) Evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( C \) is a path from \( A(0, 1, 0) \) to \( B(1, 2, 2) \).

Question 7 on next page...
7. Given the vector field \( \mathbf{F} = x^3 \mathbf{i} + y^3 \mathbf{j} \) and the closed curve \( C: x^2 + y^2 = 4 \), use Green’s theorem to do the following:

(a) (8 points) Evaluate the counterclockwise circulation \( \oint_C \mathbf{F} \cdot d\mathbf{r} \) of \( \mathbf{F} \) around \( C \).

(b) (8 points) Evaluate the outward normal flux \( \iint_C \mathbf{F} \cdot \mathbf{n} \, ds \) of \( \mathbf{F} \) across \( C \).