**Math 211: Calculus IIIA Final Exam. May 9, 2018**

1. (a) (4 points) Find the point of intersection of the line $x = 2t, y = t - 1, z = 4 - t$ and the plane $2x + y + 2z = 10$.
(b) (10 points) Find the equation of the line of intersection of the planes

$$x + y - z = 5 \quad \text{and} \quad 3x + y + z = 3.$$

2. (a) (7 points) A surface has equation $x^2 + 2xy - y^2 + z^2 = 8$. Find the equation of the tangent plane and the equation of the normal line to the surface at the point $P(2, -1, 3)$.
(b) (7 points) Find the directional derivative of $f(x, y) = x^4 - x^2y^3$ at $P(1, 1)$ in the direction of $\mathbf{u} = 4\mathbf{i} + 3\mathbf{j}$. Also, find the unit vector in the direction in which $f(x, y)$ increases most rapidly and find the rate of increase in that direction.

3. (a) (7 points) Find all local maxima, minima, and saddle points of

$$f(x, y) = x^3 + y^3 - 3xy + 4\,,$$

and evaluate $f(x, y)$ at each point.
(b) (7 points) Use the method of Lagrange multipliers to find the maximum and minimum values of $x^2 + y^2$ subject to the constraint that $x^2 - 2x + y^2 - 6y = 0$.

4. (a) (7 points) Sketch the region of integration, reverse the order of integration, and evaluate the integral

$$\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy\,.$$

(b) (7 points) A region is bounded above by the paraboloid $z = 9 - x^2 - y^2$, below by the plane $z = 0$, and lies *outside* the cylinder $x^2 + y^2 = 1$. Sketch the region. Use cylindrical polar coordinates $(r, \theta, z)$ to set up a triple integral that gives the volume of the region, then evaluate the integral to find the volume.

5. (a) (7 points) Evaluate the line integral $\int_C (y - x)\, dx$ along the path $C$: $x = t, y = 2t + 1$ for $0 \le t \le 3$.
(b) (7 points) Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ for the vector field $\mathbf{F} = 2y\mathbf{i}+3x\mathbf{j}+(x+y)\mathbf{k}$ along the curve $C$: $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + (t/6)\mathbf{k}$ from the point $A$, where $t = 0$, to the point $B$, where $t = 2\pi$.

6. Given the vector field

$$\mathbf{F} = 3x^2\mathbf{i} + \frac{z^2}{y}\mathbf{j} + (2z \ln y + z)\mathbf{k}$$

(a) (4 points) Show that the vector field $\mathbf{F}$ is conservative, or equivalently, show that it satisfies the conditions of the component test, or curl test $\nabla \times \mathbf{F} = \mathbf{0}$.
(b) (8 points) Find a scalar potential $f(x, y, z)$ for the vector field $\mathbf{F}$. Show all steps of your construction of the potential $f(x, y, z)$.
(c) (2 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $C$ is a path from $A(0, 1, 0)$ to $B(1, 2, 2)$.

**Question 7 on next page...**

7. Given the vector field $\mathbf{F} = x^3\mathbf{i} + y^3\mathbf{j}$ and the closed curve $C$: $x^2 + y^2 = 4$, use Green's theorem to do the following:
   (a) (8 points) Evaluate the counterclockwise circulation $\oint_C \mathbf{F} \cdot d\mathbf{r}$ of $\mathbf{F}$ around $C$.
   (b) (8 points) Evaluate the outward normal flux $\oint_C \mathbf{F} \cdot \mathbf{n}\, ds$ of $\mathbf{F}$ across $C$.