Math 211 Final Exam. December 20, 2017

1. (a) (12 points) Find parametric equations for the line of intersection of the planes $4x + y - 2z = 3$ and $x - 2y + z = 0$.

   (b) (8 points) Find the distance from the point P(2, 1, 1) to the line $x = 1 + 2t$, $y = -2t$, $z = -1 + t$.

2. (a) (10 points) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(1, e, 1)$ if

   $$x^2 \ln y + 2xz^3 - yz + 2e - 3 = 0\,.$$

   (b) (10 points) Use the chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ when $u = 0$ and $v = 1$ if

   $$z = \sin(xy) + x \sin y, \quad \text{and} \quad x = u^2 + v^2, \; y = uv\,.$$

3. (a) (15 points) Find the directional derivative of $f(x, y, z) = xe^y + z^2$ at $P(1, \ln 2, \frac{1}{2})$ in the direction of $\mathbf{u} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$. Also find the unit vector in the direction in which f increases most rapidly.

   (b) (15 points) Find the equation of the tangent plane and the equation of the normal line to the surface $z^2 - x^2 - 4y^2 = 0$ at the point P(3, 2, 5)

4. (a) (20 points) Find all local maxima, local minima, and saddle points of the function

   $$f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy\,.$$

   (b) (10 points) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x + 2y + 3z$ on the sphere $x^2 + y^2 + z^2 = 25$.

5. (a) (10 points) Change the Cartesian integral into an equivalent polar integral, then evaluate the polar integral, for

   $$\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2 + y^2)\, dx\, dy\,.$$

   (b) (15 points) Use cylindrical polar coordinates to find the volume of the region bounded above by the paraboloid $z = 2 - x^2 - y^2$ and below by the cone $z = \sqrt{x^2 + y^2}$.

6. (20 points) Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ of the vector field

   $$\mathbf{F} = (y + z)\mathbf{i} + (z + x)\mathbf{j} + (x + y)\mathbf{k}$$

   over the path C: $\mathbf{r} = t\mathbf{i} + t^2\mathbf{j} + t^4\mathbf{k}$ for $0 \le t \le 1$.

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7. (a) (10 points) Show that the vector field $\mathbf{F} = \sin y \cos x\,\mathbf{i} + \cos y \sin x\,\mathbf{j} + z\,\mathbf{k}$ is conservative, or equivalently satisfies the equations of the (curl) component test.

   (b) (10 points) Find a scalar potential function $f(x, y, z)$ for the vector field $\mathbf{F}(x, y, z)$, and show all steps of your construction of f.

   (c) (5 points) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is a path from the point A$(0, 1, 0)$ to B$(\frac{\pi}{2}, \frac{\pi}{2}, 2)$.

8. (30 points) Use Green's theorem to find: (a) the counterclockwise circulation, and (b) the outward flux of $\mathbf{F} = (x^2 + 4y)\mathbf{i} + (x + y^2)\mathbf{j}$ for the curve C: that bounds the square $x = 0$, $x = 1$, $y = 0$, $y = 1$.