Math 211: Calculus IIIA Common Midterm Exam 2. April 11, 2018

1. (a) (4 points) Find the gradient, ∇f, of $f(x,y,z) = \sin(xy) + y^2 - 2z^2$. Evaluate the gradient of $f(x,y,z)$ at the point $P(0,1,\frac{1}{2})$.
   (b) (10 points) Find the directional derivative of $f(x,y) = \ln y + e^x \cos x$ at $P(0,1)$ in the direction of the vector $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$. What is the direction in which $f(x,y)$ increases most rapidly at P and what is the derivative of $f(x,y)$ in this direction?

2. (a) (7 points) Find the equation of the tangent plane to the surface with equation $x^2 - xy - y^2 - z = 0$ at the point $P(1,1,-1)$.
   (b) (7 points) Use the differential $df = \nabla f \cdot \mathbf{dr}$ to find about how much the function
   $$f(x,y,z) = xz^2 - \sin(\pi xy)$$
   will change if the point $Q(x,y,z)$ moves from $Q_0(1,1,1)$ a distance $ds = \frac{1}{10}$ in the direction of $\mathbf{u} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

3. (14 points) Find all local maxima, minima, and saddle points of
   $$f(x,y) = 2x^3 + 3xy + 2y^3\,,$$
   and evaluate $f(x,y)$ at each point.

4. (14 points) An open rectangular box with no top has dimensions: length x, width y, and height z, so that its surface area is $f(x,y,z) = xy + 2xz + 2yz$ and its volume is $g(x,y,z) = xyz$. Use Lagrange multipliers to find the dimensions for which the surface area $f(x,y,z)$ is a minimum if the volume $g(x,y,z) = \frac{1}{2}$ is constant.

5. (a) (8 points) Set up and evaluate a double integral to find the area in the xy-plane bounded by $y = 0$, $x = 0$, $y = 1$, and $y = \ln x$.
   (b) (8 points) Sketch the region of integration, then reverse the order of integration, and evaluate the integral
   $$\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y}\, dy\, dx\,.$$

6. (14 points) Sketch the region of integration, then write the integral as an equivalent integral in (r,θ) polar coordinates and evaluate the polar integral for
   $$\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2 + y^2)\, dx\, dy\,.$$

7. (a) (14 points) Set up and evaluate a triple integral in (x,y,z) rectangular coordinates to find the volume of the solid (tetrahedron) in the first octant that is bounded by the coordinate planes (i.e., $x = 0$, $y = 0$, $z = 0$) and the plane $x + y + z = 1$.
   (b) **Bonus** (5 points) Use a triple integral to find the volume of the solid of part (a) if the equation of the top plane is changed to
   $$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1\,.$$