Math 211 Common Midterm Exam 2. November 15, 2017

1. (a) (10 points) Find all second order partial derivatives of

$$f(x, y) = y \sin x + 2e^{xy} + x^2 - 3xy\,.$$

(b) (10 points) Use the chain rule to express $\partial w/\partial u$ as a function of u and v if

$$w = \ln(x^2 + y^2 + z^2) \quad \text{and} \quad x = u \cos v,\ y = u \sin v,\ z = ue^v\,.$$

2. (15 points) Find the directional derivative of $f(x, y, z) = 2z^3 - 3(x^2 + y^2)z + \tan^{-1}(xz)$ at $P_0(1, 1, 1)$ in the direction of $\mathbf{u} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. Find a unit vector in the direction in which the function increases most rapidly at P_0.

3. (a) (10 points) By about how much will $f(x, y, z) = e^x \cos yz$ change as the point $P(x, y, z)$ moves from the origin a distance of $ds = 0.1$ unit in the direction of $\mathbf{i} + \mathbf{j} + \mathbf{k}$?

(b) (10 points) What is the direction of the normal to the surface $x^2 + y^2 - z^2 = 18$ at the point $P_0(3, 5, 4)$? Find the equation for the tangent plane and the normal line to the surface at P_0.

4. (15 points) Find all the local maxima, local minima, and saddle points of

$$f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8\,.$$

5. (10 points) Find the maximum and minimum values of $f(x, y) = 3x - y + 6$ subject to the constraint that the point (x, y) lies on the circle $x^2 + y^2 = 4$.

6. (a) (10 points) Sketch the region of integration, reverse the order of integration, and evaluate the integral

$$\int_0^2 \int_y^2 e^{x^2}\, dx dy\,.$$

(b) (10 points) Sketch the region bounded by the parabola $x = y - y^2$ and the line $y = -x$. Then express the region's area as a double integral and evaluate the integral.