Math 211: Calculus IIIA Common Midterm Exam 1. February 28, 2018

1. Given the vectors $\mathbf{u} = \mathbf{i}+\mathbf{j}+2\mathbf{k} = \langle 1, 1, 2\rangle$, $\mathbf{v} = 2\mathbf{i}-\mathbf{j}+\mathbf{k} = \langle 2, -1, 1\rangle$, and $\mathbf{w} = \mathbf{i}+3\mathbf{k} = \langle 1, 0, 3\rangle$ find:
   (a) (3 points) The dot product $\mathbf{u} \cdot \mathbf{v}$
   (b) (3 points) The cross product $\mathbf{v} \times \mathbf{w}$
   (c) (3 points) The projection of $\mathbf{u}$ onto $\mathbf{v}$, $\text{proj}_{\mathbf{v}}\mathbf{u}$
   (d) (3 points) The volume of the parallelepiped formed on $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

2. (a) (8 points) Are the two lines $l_1 : x = 2+t, y = 2+3t, z = 3+t$ and $l_2 : x = -1+2s, y = -2+s, z = 1+s$ parallel, do they intersect, or are they skew? If they intersect give their point of intersection. (Explain your answer.)
   (b) (8 points) Find the equation of the plane that contains the line $l : x = 2+t, y = 1-t, z = 3+t$ and the point $P_0(1, 0, 1)$.

3. A curve has parametric equation $\mathbf{r} = t\,\mathbf{i} + 2t\,\mathbf{j} + \frac{2}{3}t^{3/2}\,\mathbf{k} = \langle t, 2t, \frac{2}{3}t^{3/2}\rangle$.
   (a) (6 points) Find the unit tangent vector to the curve at the point where $t = 1$.
   (b) (4 points) Find the equation of the tangent line to the curve at the point where $t = 1$.
   (c) (5 points) Find $\frac{ds}{dt}$ where s is arc length along the curve. Find the total arc length s along the curve from the point where $t = 0$ to the point where $t = 4$.

4. (14 points) Solve the initial value problem to find $\mathbf{r} = \mathbf{r}(t)$ for all $t \geq 0$, when
$$\frac{d\mathbf{r}}{dt} = \frac{3}{2}(t+1)^{1/2}\mathbf{i} + e^{-t}\mathbf{j} + \frac{1}{t+1}\mathbf{k}$$
   with initial condition $\mathbf{r}(0) = \mathbf{k}$.

5. (16 points) Find all first and second order partial derivatives of
$$f(x, y) = x^3y^2 + \cos(xy) + y\sin x$$

6. (15 points) If $w = xy + \ln z$ with
$$x = \frac{v^2}{u}, \quad y = u + v, \quad z = \cos u,$$
   use the chain rule to find the partial derivatives $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at the point where $(u, v) = (-1, 2)$.

7. (12 points) Given the function
$$f(x, y) = \ln(x^2 + y^2 - 1)$$
   Find the function's domain and range. Describe or sketch the function's level curves. Find the boundary of the function's domain. Is the domain open, closed, or neither (explain)? Is the domain bounded or unbounded (explain)?