Math 211, Common Midterm Exam 1. October 11, 2017

1. Given the vectors $\mathbf{u} = 3\,\mathbf{i} - \mathbf{k}$, $\mathbf{v} = \mathbf{i} + 2\,\mathbf{j} + 2\,\mathbf{k}$, and $\mathbf{w} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$, find:
   (a) (3 points) $\mathbf{u} \cdot \mathbf{v}$.
   (b) (3 points) The scalar component of $\mathbf{u}$ in the direction of $\mathbf{v}$, $\text{comp}_{\mathbf{v}}\mathbf{u}$.
   (c) (3 points) $\mathbf{v} \times \mathbf{w}$.
   (d) (3 points) The volume of the parallelepiped formed on $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

2. Given the points $P(1, 2, 1)$, $Q(2, 3, 4)$ and $R(-1, 2, -1)$ in space,
   (a) (8 points) Find the area of the parallelogram formed with two sides PQ and PR.
   (b) (10 points) The equation of the plane through the points P, Q, and R.

3. (a) (10 points) Find the point where the line through $P(2, 1, 1)$ and $Q(3, 3, 0)$ intersects the plane $3x - y + 2z = 8$.
   (b) (10 points) Given the two lines with equations

$$\begin{aligned} &\text{L1:} \quad x = -1 + 2t, \quad y = 1 - t, \quad z = 1 + 2t, \\ &\text{L2:} \quad x = 1, \quad y = 6 + 3r, \quad z = 5 + r, \end{aligned}$$

   are they parallel, are they skew, or do they intersect? Explain, and if they intersect give the point of intersection.

4. The position vector of a particle moving through space is

$$\mathbf{r}(t) = (2\cos t)\,\mathbf{i} + (2\sin t)\,\mathbf{j} + t\,\mathbf{k}$$

   (a) (8 points) Find the velocity and acceleration vectors. Are they orthogonal?
   (b) (8 points) If s is the arc length along the path, find $\frac{ds}{dt}$ and the arc length of the portion of the curve for $0 \le t \le \frac{\pi}{2}$.
   (c) (6 points) Find the parametric equations of the tangent line to the path at the point where $t = \frac{\pi}{2}$.

5. (16 points) Find the position vector $\mathbf{r}(t)$ for a particle moving in space with

$$\frac{d\mathbf{r}}{dt} = \frac{3}{2}(t+1)^{1/2}\,\mathbf{i} + e^{-t}\,\mathbf{j} + \frac{1}{t+1}\,\mathbf{k}\,, \quad \text{and } \mathbf{r}(0) = \mathbf{k}\,.$$

6. Given the function of two variables

$$f(x, y) = 9x^2 + 4y^2$$

   (a) (6 points) what is the domain of $f(x, y)$, what is the range of $f(x, y)$, and are they bounded or unbounded?
   (b) (6 points) Sketch three level curves of $f(x, y)$ in the xy-plane, and use these to sketch the surface $z = 9x^2 + 4y^2$ in (x, y, z) space.