1. Determine if the following series converges or diverges. If it converges, find its sum. Clearly show all work, including any applicable tests for convergence. \( (6 \text{ points}) \)

\[
\sum_{n=2}^{\infty} 5 \left( \frac{-1}{3} \right)^n
\]
2. Determine if the following series converge or diverge. If they converge, find their sum. Clearly show all work, including any applicable tests for convergence. (12 points)

\[ a. \sum_{n=0}^{\infty} \frac{3^{n+1}}{2^{2n}} \]

\[ b. \sum_{n=3}^{\infty} \frac{n^2}{(n+2)(n-2)} \]
3. Determine if the following series converge or diverge. Clearly show all work, including any applicable tests for convergence. (16 points)

\[ a. \sum_{n=1}^{\infty} \frac{2n^2}{3 \sqrt[3]{n^7 + n}} \]

The quality of the notation, logic, organization, and/or neatness on this problem is (circle one):
- 1 point (unacceptable)
0 points (acceptable)
+1 point (superior)

\[ b. \sum_{n=1}^{\infty} \frac{n}{e^{-n} + n^3} \]

The quality of the notation, logic, organization, and/or neatness on this problem is (circle one):
- 1 point (unacceptable)
0 points (acceptable)
+1 point (superior)
4. Determine if the series converges conditionally, converges absolutely or diverges. Clearly show all work, including any applicable tests for convergence. (10 points)

\[ \sum_{n=2}^{\infty} \frac{(-1)^n}{n \sqrt{\ln(n)}} \]

The quality of the notation, logic, organization, and/or neatness on this problem is (circle one):
- 1 point (unacceptable)
- 0 points (acceptable)
+ 1 point (superior)

5. Suppose that \( f(n) > 0 \) for all integers \( n \) and that the series \( \sum_{n=0}^{\infty} f(n) \) is known to be convergent. Is the above series absolutely convergent, conditionally convergent, divergent, or is it unable to determine? Carefully explain how you know, or what necessary information you are missing. (4 points)
6. Find the radius of convergence and interval of convergence for the power series
\[ \sum_{n=1}^{\infty} \frac{(x - 1)^n}{n3^n} \quad (12 \text{ points}) \]
7. Find the Taylor Polynomial $P_3(x)$ that uses a Taylor Series centered around $x=3$ to approximate $f(x)=\ln(x-2)$ (5 points)

b. Find the series representation of the full Taylor Series for the function above (6 points)
8. Find the full Taylor Series representation for \( f(x) = e^{-x/2} \) centered around \( x=1 \) (7 points)

b. Find the Radius of Convergence and Interval of Convergence for this Taylor Series by performing an appropriate convergence test on the power series above. (7 points)
9. Find a Maclaurin Series for the function \( f(x) = \cos(x^2) \) (hint: the substitution method using a well-known Maclaurin series will be the quickest) \( (5 \text{ points}) \)

b. Find a Maclaurin series representation for the indefinite integral \( \int \cos(x^2) \, dx \) \( (5 \text{ points}) \)

c) Estimate the integral \( \int_0^1 \cos(x^2) \, dx \) with an error of magnitude less than 0.01 by using a Taylor Polynomial of the lowest possible degree. \( (5 \text{ points}) \)