Math 111 Final Exam
December 15, 2017

Time: 2 hours and 30 minutes
Instructions: Show all work for full credit.
No outside materials or calculators allowed.
Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: ______________________________

ID #: ______________________________

Instructor/Section: ___________________

"I pledge by my honor that I have abided by the NJIT Academic Integrity Code."

__________________________ (Signature)

Problem	Value	Score
1	10 pts.	
2	10 pts.	
3	15 pts.	
4	10 pts.	
5	15 pts.	
6	10 pts.	
7	15 pts.	
8	15 pts.	
TOTAL	100	

1. Consider the function $y = f(x) = |x^2 - 1|$.

(a) Sketch *f*, noting that $x^2 - 1$ is negative for $-1 < x < 1$. (Hint: Plot $y = x^2 - 1$.) **(3 pts.)**

(b) Show that the function is continuous everywhere. **(2 pts.)**

(c) Show $f'(x)$ is not continuous at $x = 1$ by showing $\lim_{x\to 1-} f'(x) \neq \lim_{x\to 1+} f'(x)$. **(5 pts.)**

2. Let $y = y(x)$ be implicitly defined by $y = 1 + e^x \sin(xy)$.

(a) Compute the derivative *dy*/*dx*. **(5 pts.)**

(b) Find the tangent line to the curve at $(\pi, 1)$. . **(5 pts.)**

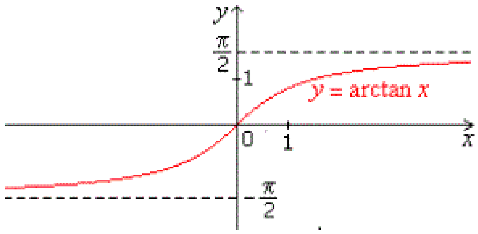
3. (a) A rectangular yard is to be completely enclosed by fencing and then divided into three enclosures of equal area by fences parallel to and of the same length as one side of the yard. If 400 ft. of fencing is available, what dimensions maximize the enclosed area? Verify that your answer is the largest possible area. **(8 pts)**

(b) Find a positive number for which the sum of its reciprocal and four times its square is the smallest possible. Verify that your answer is the smallest possible sum. **(7 pts.)**

4. Consider the function $y = f(x) = \dfrac{\arctan x}{1+x^2} = \dfrac{\tan^{-1} x}{1+x^2}$.

(a) Use the derivative to show that the function has its maximum and minimum at the positive and negative solutions, respectively, of $\tan\left(1/(2x)\right) = x$, which happen to be at $x \simeq \pm 0.765$. Show your work and state any tests used. **(7 pts.)**

(b) Sketch the graph of the function using the definition and the above information. (Hint: the graph of arctan$x$ shown below should be helpful here and for (a)). **(3 pts.)**



5. Consider the function $y = f(x) = \dfrac{x^2}{1+x^2}$.

(a) Find and classify (as local or global maxima or minima) any critical points. **(3 pts.)**
(b) Find all asymptotes of the curve. **(3 pts.)**
(c) Find all inflection points. **(3 pts.)**
(d) Find where the function is increasing and decreasing. **(3 pts.)**
(e) Sketch the curve using the above information. **(3 pts.)**

6. Evaluate each of the following limits:

(a) $\lim_{x\to 0} \dfrac{\tan x}{\arctan x} = \lim_{x\to 0} \dfrac{\tan x}{\tan^{-1} x}$. **(3 pts.)**

(b) $\lim_{x\to\infty} (1+2x)^{1/(2\ln x)}$. **(4 pts.)**

(c) $\lim_{x\to 0+} x^x$. **(3 pts.)**

7. The function $f(x)$ is differentiable. In the table below are some values of $f(x)$ and its integrable derivative $f'(x)$.

x	1	2	3	4	5	6
$f(x)$	3	2	0	1	4	5
$f'(x)$	-3	-2	1	2	3	4

a) Approximate $\int_0^6 f(x)dx$ by a Riemann sum using the midpoint rule with three rectangles. **(5 pts.)**

b) Approximate $\int_0^6 f(x)dx$ by a Riemann sum by using right endpoints and six rectangles. **(5 pts.)**

c) Find the exact value of $\int_1^4 f'(x)dx$. Show your work and state any theorems that you use. **(5 pts.)**

8. Solve each of the following:

(a) Evaluate $\int_0^{\pi^2} \frac{\cos(\sqrt{x})dx}{\sqrt{x}}$. **(3 pts.)**

(b) Evaluate $\int \left(xe^{x^2} + \frac{1}{x \ln x} \right) dx$ **(3 pts.)**

(c) Evaluate $\int_0^{\pi} \tan x \cos^2 x dx$. **(3 pts.)**

(d) Evaluate $2\int_0^1 (2-2x^2)dx$. **(2 pts)**

(e) Show that (d) is the area between the curves $y = -x^2$ and $y + 3x^2 = 2$. **(4 pts.)**