Math 110 Common Exam #1
September 27, 2017

Time: 1 hour and 25 minutes

Instructions: Show all work for full credit. No outside materials or calculators allowed.

Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

<table>
<thead>
<tr>
<th>Problem(s)</th>
<th>Score</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Name: _______________________________

ID #: _______________________________

Instructor/Section: _______________________________

"I pledge by my honor that I have abided by the NJIT Academic Integrity Code."

_________________________ (Signature)

Relevant Formulas for this Exam:

Circular motion and equations relating to a sector of a circle, radius r (as shown to the right).

\[ s = r\theta \]

\[ v = r\omega \]

\[ A = \frac{1}{2} r^2 \theta \]

\[ P = P_0 e^{kt} \]
1. (16 pts) Find the exact value of the following expression.
   a) \(2 \ln \sqrt{2} + 2 \log_2 4 + \log_2 16\)
   \[
   2 \ln \sqrt{2} + 2 \log_2 4 + \log_2 16 \\
   2(\frac{1}{2}) + 4 + 4 \\
   1 + 4 + 4 \\
   9
   \]
   b) \(\log_3 135 - \log_3 45\)
   \[
   \log_3 (\frac{135}{45}) = \log_3 3 \\
   = 1
   \]
   c) \(\log_2 [e^{\ln 8}]\)
   \[
   \log_2 8 \\
   \frac{\log_2 2^3}{3} \\
   \]
   d) \(2[\cos(45^\circ)]^2 + 2[\sin(45^\circ)]^2 - 2 \tan(45^\circ)\)
   \[
   2 \cos^2 45^\circ + 2 \sin^2 45^\circ - 2 \tan 45^\circ \\
   2(1) - 2(1) \\
   2 - 2 \\
   0
   \]

2. (6 pts) Given that \(\cos \theta = \frac{2}{5}\), where \(\theta\) is an acute angle, find the exact values of the 5 other trigonometric functions. (Rationalize any and all denominators).

   \[
   y = \sqrt{5^2 - 2^2} = \sqrt{25 - 4} = \sqrt{21}
   \]

   \[
   \cos \theta = \frac{2}{5}
   \]

   \[
   \sec \theta = \frac{5}{2}
   \]

   \[
   \sin \theta = \frac{\sqrt{21}}{5}
   \]

   \[
   \csc \theta = \frac{5}{\sqrt{21}}
   \]

   \[
   \tan \theta = \frac{\sqrt{21}}{2}
   \]

   \[
   \cot \theta = \frac{2}{\sqrt{21}}
   \]

   \[
   = \frac{2\sqrt{21}}{21}
   \]
3. a) (4 pts) Write the expression in condensed (compressed) form with a coefficient of 1. (Assume all variables represent positive numbers)

\[
\frac{1}{3}\log(x + 2)^3 + \frac{1}{2}\left[\log(x^4) - \log(x^2 - x - 6)^2\right] \\
\frac{1}{3}\log(x+2) + \frac{1}{2}\left[\log\left(\frac{x^4}{(x^2-x-6)^2}\right)\right] \\
\log(x+2) + \log\left[\frac{x^4}{(x^2-x-6)^2}\right] \\
\log\left[\frac{(x+2)x^2}{x^2-x-6}\right] = \log\left[\frac{(x+2)x^2}{(x-3)(x+2)}\right] = \log\left[\frac{x^2}{x-3}\right]
\]

b) (4 pts) Write the expression in expanded form. (Assume all variables represent positive numbers)

\[
\log\sqrt{100x\sqrt{y}} = \log\left[\frac{100x\sqrt{y}}{\sqrt{2}}\right] \\
= \frac{1}{2}\log\left[100x \cdot y^{\frac{1}{2}}\right] \\
= \frac{1}{2}\left[\log 100 + \log x + \log y^{\frac{1}{2}}\right] \\
= \frac{1}{2}\left[\log 10^2 + \log x + \frac{1}{2}\log y\right] \\
= \frac{1}{2}\left[2 + \log x + \frac{1}{2}\log y\right] \\
= \frac{1}{2}(2) + \frac{1}{2}\log x + \frac{1}{2}\cdot\frac{1}{2}\log y = 1 + \frac{1}{2}\log x + \frac{1}{4}\log y
\]

4. (10 pts) Graph the function \( y = -2^{x^2} - 1 \), on the set of axes below by making a table of values or using transformation. Be sure to label the asymptote on the graph, if any exists.

Using Table of Values

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-2.3</td>
</tr>
<tr>
<td>-2</td>
<td>-2.0</td>
</tr>
<tr>
<td>-1</td>
<td>-3.0</td>
</tr>
</tbody>
</table>

Using Transformation

\[
y' = 2^x \quad \text{shifts left by 2 units} \\
x' = y \quad \text{reflect about x-axis} \\
y' = 2^{x+2} \quad \text{vertical shift down 1 unit} \\
\]

Using Transformation

\[
y' = 2^x \quad (a) \\
y' = 2^{x+2} \quad (b) \\
y' = -2^{x+2} \quad (c) \\
y' = -2^x \quad (d)
\]

\[
y = -2^{x^2} - 1 \\
y = 2^x \\
y = -2^x \\
y = -2^{x+2} \\
\]
5. Given the 2 in. radius wheel and 7 in. radius wheel pulley system as shown below, find the following. *(You Can't Use Ratios).*

**8 in. wheel**

\[ r_a = 2 \text{ in.} \]
\[ \theta_a = \frac{5\pi}{3} \text{ deg} \]
\[ = \frac{5\pi}{18} \text{ rad} \]

\[ \delta_2 = r_a \theta_a \]
\[ = 2 \left( \frac{5\pi}{18} \right) \]
\[ = \frac{5\pi}{9} \text{ in.} \]

**7 in. wheel**

\[ r = 7 \text{ in.} \]
\[ \delta_7 = \frac{5\pi}{2} \text{ in.} \]
\[ \theta_7 = \frac{\delta_7}{r} = \frac{5\pi}{14} = \frac{5\pi}{28} \text{ rad} \]
\[ \theta_7 \text{ deg} = \frac{5\pi}{28} \text{ rad} \left( \frac{180 \text{ deg}}{\pi \text{ rad}} \right) \]
\[ = 160 \text{ deg} \]

a) *(5 pts)* If the 2 in. radius wheel turns through an angle of 50°, what angle (expressed in degrees) does the 7 in. radius wheel turns through.

b) *(5 pts)* If the 2 in. radius wheel is spinning at a rate of 6 rpms, how many rpms is the 7 in. radius wheel making?

**8 in. wheel**

\[ r_a = 2 \text{ in.} \]
\[ \omega_2 = \frac{6 \text{ rev}}{\text{min}} \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 12\pi \text{ rad/min} \]
\[ v_2 = r_a \omega_2 \]
\[ = 2 \left( 12\pi \right) = 24\pi \text{ in/min} \]

**7 in. wheel**

\[ r_7 = 7 \text{ in.} \]
\[ v_7 = v_2 = 24\pi \text{ in/min} \]
\[ \omega_7 = \frac{v_7}{r_7} = \frac{24\pi}{7} \text{ rad/min} \]
\[ \omega \text{ rpm} = \frac{24\pi \text{ rad/min}}{\frac{2\pi \text{ rad}}{1 \text{ rev}}} \]
\[ = \frac{12}{7} \text{ rpm} \]
6. a) (5 pts) Find the height of a tree that casts a 100-feet shadow on the ground if \( \theta \) (the angle of elevation) to the sun from the ground is 45° as shown in the diagram below.

\[
\tan 45^\circ = \frac{h}{100}
\]

\[
h = 100 \tan 45^\circ \\
= 100(1) \\
= 100 \text{ ft}
\]

b) (5 pts) Given the right triangle as labeled below, if \( \sin \theta = \frac{1}{4} \), find the lengths of side ‘a’ and ‘c’. Simplify your answers as much as possible.

Using similar triangles,

\[
since \sin \theta = \frac{1}{4} = \frac{1}{c},
\]

then \( c = 8 \)

Therefore, \( a^2 = 8^2 - 2^2 \)

\[= 64 - 4 \]

\[= 60\]

\[a = \pm 2\sqrt{15}\]

\[a = 2\sqrt{15}\]
7. (20 pts) Solve the following equation for all real solutions. Make sure all answers are in the domain of the original problem.

a) \( \log_6(x + 2) + \log_6(x - 3) = 1 \)

\[
\log_6 [(x+2)(x-3)] = 1 \\
(x+2)(x-3) = 6 \\
x^2 - x - 6 = 6 \\
x^2 - x - 12 = 0 \\
(x-4)(x+3) = 0 \\
x = 4 \quad \text{or} \quad x = -3
\]

\[\text{Don't work in arguments of original equations}\]

\( \text{\&} \)

b) \( 3xe^x + x^2e^x = 0 \)

\[
x e^x (x+3) = 0 \\
x = 0 \quad e^x = 0 \\
x = -3
\]

\( e^x \) can never be equal to 0

\[\{0, -3\}\]

c) \( 4(2^{x-2}) - 5 = 27 \)

\[
4 \cdot 2^{x-2} = 32 \\
2^{x-2} = 8 = 2^3 \\
6 - 2x = 3 \\
-2x = -3 \\
x = \frac{3}{2}
\]

\[\{3/2\}\]

d) \( \log(x^2 + 1) = \log(x - 2) + \log(x + 3) \)

\[
\log(x^2 + 1) = \log [(x-2)(x+3)] \\
\text{Using one-to-one property:} \\
x^2 + 1 = (x-2)(x+3) \\
x^2 + 1 = x^2 + x - 6 \\
x^2 + 1 - x - 6 = 0 \\
17 - x = 0 \\
x = 17
\]

\[\{17\}\]
8. a) (5 pts) Suppose that $\cos \theta = \frac{1}{x}$ where 'x' is a nonzero constant. Find the values of the other 5 trigonometric functions in terms of 'x'. (You do not need to rationalize the denominator)

\[
\begin{align*}
\cos \theta &= \frac{1}{x} \\
\sec \theta &= x \\
\sin \theta &= \frac{\sqrt{x^2 - 1}}{x} \\
\csc \theta &= \frac{x}{\sqrt{x^2 - 1}} \\
\tan \theta &= \frac{\sqrt{x^2 - 1}}{1} \\
\cot \theta &= \frac{1}{\sqrt{x^2 - 1}} \\
&= \sqrt{x^2 - 1}
\end{align*}
\]

b) (5 pts) Given the rectangle (4 feet by 2 feet) inscribed in the semicircle as shown below, find the area of the shaded region of the semicircle.

Area of Rectangle = 4 \cdot 2 = 8 \text{ ft}^2 \\
Radius of semicircle = 2\sqrt{2} \\
Area of semicircle = \frac{1}{2} \pi r^2 \\
&= \frac{1}{2} \pi (2\sqrt{2})^2 \\
&= \frac{1}{2} \pi (8) \\
&= 4\pi \text{ ft}^2 \\
Area of shaded region of semicircle &= Area of semicircle - Area of rectangle \\
&= (4\pi - 8) \text{ ft}^2 \\
&= 4(\pi - 2) \text{ ft}^2
9. a) (4 pts) Given that \( f(x) = 5^x + 5^{-x} \) and \( g(x) = 5^x - 5^{-x} \), evaluate the following expression below.  

\[ \left[ f(x) \right]^2 + \left[ g(x) \right]^2 \]

\[
\left[5^x + 5^{-x}\right]^2 = 5^{2x} + 2 \cdot 5^x \cdot 5^{-x} + 5^{-2x} = 5^{2x} + 2 + 5^{-2x}
\]

\[
\left[5^x - 5^{-x}\right]^2 = 5^{2x} - 2 \cdot 5^x \cdot 5^{-x} + 5^{-2x} = 5^{2x} - 2 + 5^{-2x}
\]

Therefore, \( \left[ f(x) \right]^2 + \left[ g(x) \right]^2 = (5^{2x} + 2 + 5^{-2x}) + (5^{2x} - 2 + 5^{-2x}) = 2 \cdot 5^{2x} + 2 \cdot 5^{-2x} = 2 \left[ 5^{2x} + 5^{-2x} \right] \)

b) (3 pts) Simplify completely (give your answer with positive exponents only)

\[
\left[ \frac{(-x^2y)^3}{(xy)^5} \right]^{-2} = \left[ \frac{-x^6y^3}{x^5y^5} \right]^{-2} = \left[ \frac{-x}{y} \right]^{-2} = \left[ \frac{y^2}{x} \right]^{-2} = \frac{y^4}{x^2}
\]

c) (3 pts) Simplify completely.

\[
\frac{\sqrt{16-16x^2}}{4-4x} = \frac{\sqrt{16(1-x^2)}}{4(1-x)}
\]

\[
= \frac{4 \sqrt{1-x^2}}{4(1-x)}
\]

\[
= \frac{\sqrt{1-x^2}}{1-x}
\]