Math 110 Common Exam II
October 26, 2016

Time: 1 hour and 25 minutes

Instructions: Show all work for full credit.
No outside materials or calculators allowed.

Extra Space: Use the backs of each sheet
for extra space. Clearly label when doing so.

Name: _____

ID #: _____

Instructor/Section: _____

*"I pledge by my honor that I will abide by the
NJIT Academic Integrity Code."*

_____ (Signature)

Problem(s)	Score	Total

Relevant Formulas for this Exam

$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
$\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b)$
$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

Rationalize any and all denominators, if necessary.

1. a) **(8 pts)** Graph <u>two periods</u> of the trigonometric function below. Clearly label all asymptotes and zeros.
$$y = \tan(3x - \pi)$$

b) **(8 pts)** Find the amplitude, the period and the phase shift of the trigonometric function below. Then graph one period of the function. Clearly label all quarter points.
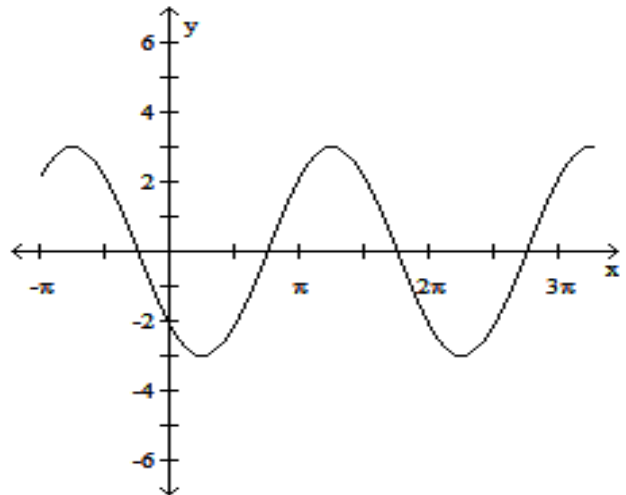$$y = -2\cos\left(\frac{\pi}{3}x\right) + 1$$

2. (**6 pts**) Evaluate the following.

   a) $\tan\left(\sin^{-1}\left(\dfrac{3}{7}\right)\right)$

   b) $\sin\left[\sin^{-1}\left(\dfrac{1}{2}\right) - \cos^{-1}\left(-\dfrac{1}{2}\right)\right]$

3. (**7 pts**) The graph below is a graph of a cosine wave of the general form: $y = a\cos b(x - c) + d$ where a, b, c and d are constants. Find possible values of a, b, c and d that satisfy the equation.

4. (12 pts) Find the exact value of the following. *Draw a quadrant diagram, if necessary.*

a) $\tan\left(-\frac{17\pi}{4}\right)$

b) $\sin\left(\frac{5\pi}{12}\right)$

c) $\cos(112.5°)$

5 (**5 pts**) Verify the identity: $\dfrac{\sin x}{1+\cos x} = \csc x - \cot x$

6. a) (**4 pts**) Given that $\theta = \dfrac{41\pi}{9}$, determine the quadrant θ lies in and find its reference angle.

b) (**7 pts**) Suppose θ is an angle whose terminal side contains the point $P(-2,5)$. Find the exact values of the six trigonometric functions.

7. (**8 pts**) The average temperatures in a certain region is given by the function $N(t) = 45 + 25 \sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right)$, with t given in months and $t = 1$ represents the month of January.

   a) Find the period of the function $N(t)$.

   b) Find the maximum and minimum average temperatures in the region at any given time and find the months they will occur.

   c) What is the average temperature for the month of April?

8. **(15 pts)** Given that $\cos\theta = \dfrac{k}{4}$, with angle θ in quadrant I. Assume 'k' is a positive constant. Find the value of following trigonometric expression in terms of 'k'.

a) $1 - \sin^2\theta$

b) $\sin(-\theta)$

c) $\cos(\theta + 8\pi)$

d) $4\cos^2\theta - 4\sin^2\theta$

e) $10\tan^2\theta - 10\sec^2\theta$

9. **(10 pts)** If $\cos\alpha = -\dfrac{4}{5}$, with α in Quad. III and $\sin\beta = \dfrac{5}{13}$, with β in Quad. II, find the following.

a) $\sin(\alpha - \beta)$

b) $\cos(\alpha - \beta)$

c) $\tan(\alpha - \beta)$

d) In what quadrant will angle $(\alpha - \beta)$ lie? Carefully explain or justify your answer.

10. **(10 pts)** A wheel has a radius of 3 inches. A point on the wheel has initial coordinates of $\left(-\sqrt{5}, 2\right)$ relative to the center of the wheel. Find the coordinates of this point relative to the center of the wheel after the wheel rolls 20π inches to the left.

Solution.

Check: $\left(-\sqrt{5}\right)^2 + 2^2 = 5 + 4 = 9 = 3^2$, so the point lies on the wheel.

Circumference $= 2\pi(3) = 6\pi$.

Rolling 20π inches corresponds to a rotation angle of
$$\theta = \frac{20\pi}{3} = 6\pi + \frac{2\pi}{3}.$$

Net rotation: $\dfrac{2\pi}{3}$ counterclockwise (rolling left).

With $\cos\frac{2\pi}{3} = -\frac{1}{2}$, $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$:

$$x' = (-\sqrt{5})\left(-\tfrac{1}{2}\right) - 2\left(\tfrac{\sqrt{3}}{2}\right) = \frac{\sqrt{5}}{2} - \sqrt{3}$$

$$y' = (-\sqrt{5})\left(\tfrac{\sqrt{3}}{2}\right) + 2\left(-\tfrac{1}{2}\right) = -\frac{\sqrt{15}}{2} - 1$$

$$\boxed{\left(\frac{\sqrt{5}}{2} - \sqrt{3},\ -\frac{\sqrt{15}}{2} - 1\right) \approx (-0.61,\ -2.94)}$$