

DOCTORAL QUALIFYING EXAM
New Jersey Institute of Technology
Department of Mathematical Sciences

Applied Math Part A: Applied Math

August 2020

These three questions are based on Math 651.

1. (a) Find the general solution to the ODE $t^2 u_{tt} + 2tu_t = \log(t)$ for $t > 1$ with $u(1) = 0 = u_t(1)$. Simplify your answer.
(b) Consider the first order PDE $u_t + 4u_x = 1$ with $u(x, 0) = \tan(x)$. Use the method of characteristics to solve it and simplify the explicit form of $u(x, t)$. Explain your steps and verify the solution.
2. Consider the system $x_t = 1 - xy$ and $y_t = (x - 1)y$.
 - (a) Is the system Hamiltonian? If so, find the Hamiltonian. If not, explain why.
 - (b) Find the fixed points and determine their nature. Indicate any stable or unstable directions.
 - (c) Find the nullclines.
 - (d) Draw the phase portrait where fixed points, trajectories, nullclines, stable and unstable directions are clearly indicated.
3. Suppose that for $0 < x < 1$

$$u_t = u_{xx} + e^t$$

subject to boundary conditions $u(0, t) = 0 = u(1, t)$, and initial condition $u(x, 0) = x(1 - x)$. Find $u(x, t)$ and explain all the steps.