Ph.D. Prelim: Exam. B
Statistical Inference

May 16, 2018

1. Let g be a one-to-one function. Prove that the maximum likelihood estimator (MLE) of $g(\theta)$ is given by $g(\hat{\theta})$, where $\hat{\theta}$ is the MLE of θ.

2. This section has two independent questions.

(i) Let $X_1, X_2, \ldots, X_n$ denote a random sample from the distribution

$$f(x;\theta) = \begin{cases} \left(\frac{\theta}{2}\right)^{|x|} (1-\theta)^{1-|x|} & x = -1, 0, 1, \qquad 0 \le \theta \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find the minimum variance unbiased estimator (MVUE) of $P(X_1 = 1)$.

(ii) Let $X_1, X_2, \ldots, X_n$ denote a random sample from the $\mathrm{N}(\theta, 1)$ distribution. An initial estimator of θ is X_1. Derive the MVUE using conditioning on the initial estimator.

3. Suppose that $X_1, X_2, \ldots X_{10}$ denote a random sample from a distribution with pdf

$$f(x;\theta) = \frac{1}{\theta} \exp\left(-\frac{x}{\theta}\right) I_{(0,\infty)}(x), \qquad \theta > 0.$$

Derive the level 0.05 likelihood ratio test for testing $H_0 : \theta = 2$ against $H_1 : \theta \neq 2$. What is the power of the test when $H_1 : \theta = 1.5$?