Math 112 FINAL EXAM, Spring, 2022

Read each problem carefully. Show all your work for each problem! Use only those methods discussed thus far in class. No Calculators!

1. (10) Evaluate the following integrals:

$$(a)\ \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx, \qquad (b)\ \int \frac{2\,dx}{x^2+2x}.$$

2. (10) Evaluate the following integrals:

$$(a)\ \int xe^{-x}\,dx, \qquad (b)\ \int \sec^2 x\,\tan^2 x\,dx.$$

3. (a) (6) Use the Disk method to find the volume of the solid generated by revolving the region bounded between the curves $y = \frac{6}{x^{3/2}}$, $y = 0$, $x = 1$ and $x = 3$ about the x-axis.

   (b) (6) Find the length of the curve $y = 1 + (x - \frac{4}{9})^{3/2}$ from $x = 1$ to $x = 4$.

4. (10) Evaluate the following integrals:

$$(a)\ \int (1+\sin^2 x)\,dx \qquad (b)\ \int \frac{1-x}{1+4x^2}\,dx.$$

5. (a) (5) Estimate the error if e^{-x^2} is approximated by $1 - x^2 + \frac{x^4}{2!}$ in the integral

$$\int_0^1 e^{-x^2}\,dx.$$

   (b) (5) Find the terms in the Maclaurin series up to, and including, x^3, for the following function (you may use known series):

$$f(x) = \frac{\sin 2x}{1-x}.$$

6. (10) Use the **integral test** in (a) and **a comparison test** in (b) to determine whether the series converge or diverge:

$$(a)\ \sum_{n=2}^{\infty} \frac{1}{n\ln n} \qquad (b)\ \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{1}{2}\right)^n.$$

7. (10) Use the **ratio test** in (a) and the **root test** in (b) to determine whether the series converge or diverge:

$$(a)\ \sum_{n=1}^{\infty} \frac{n!}{e^{n^2}} \qquad (b)\ \sum_{n=3}^{\infty} \left(\frac{\ln n^2}{n}\right)^n.$$

8. (a) (5) Determine whether the following series converges. State which test you use.

$$\sum_{n=0}^{\infty} \frac{1+n^2}{1+10n^2}.$$

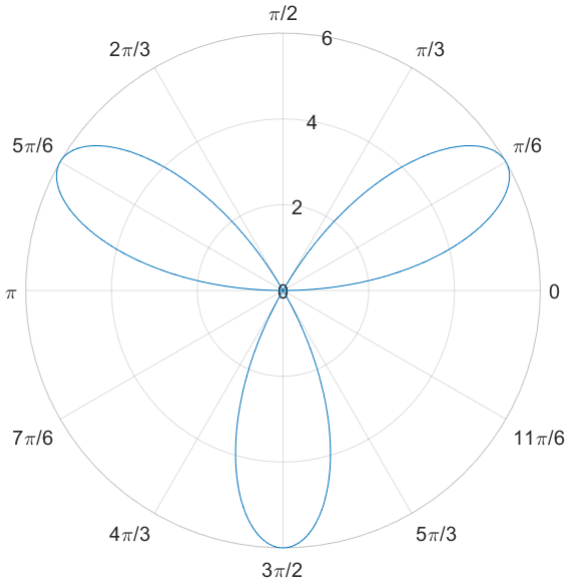
(b) (5) Determine the radius of convergence and the interval of convergence for the power series:

$$\sum_{n=1}^{\infty} \frac{(2x)^n}{n^3}.$$

9. (10) Evaluate the following integrals:

$$(a) \int_0^1 \frac{1}{\sqrt{1-x}}\,dx, \qquad (b) \int \frac{x^3}{\sqrt{1-x^2}}\,dx.$$

10. (a) (8) Find the area of the polar region that lies inside one loop of the curve $r = 6\sin 3\theta$.



(b) (6) Find an equation for the line tangent to the curve $x = e^t$, $y = (t-1)^2$ at the point $t = 0$.