

### Math 112 Exam 3, Spring 2022

Read each problem carefully. Show all your work for each problem! Be sure to identify any tests used to determine convergence or divergence.

1. (12) Determine whether the following series are convergent or divergent. Please state which test you are using.

$$(a) \sum_{n=1}^{\infty} \left( \frac{2+n^2}{1+2n^2} \right), \quad (b) \sum_{n=1}^{\infty} \frac{1}{n^2 + \sqrt{n}}$$

2. (12) Sum the following series:

$$(a) \sum_{n=1}^{\infty} \frac{1}{n(n+1)}, \quad (b) \sum_{n=0}^{\infty} \frac{2^{2n+1} - 6^n}{8^n}.$$

3. (12) Use the **integral test** in (a) and a **comparison test** in (b) to determine whether the series converge or diverge:

$$(a) \sum_{n=3}^{\infty} \frac{1}{n(\ln n + 1)^2} \quad (b) \sum_{n=1}^{\infty} \left( \frac{1}{n} + \left( \frac{1}{2} \right)^n \right).$$

4. (12) Use the **ratio test** in (a) and the **root test** in (b) to determine whether the series converge or diverge:

$$(a) \sum_{n=1}^{\infty} \frac{e^{n^2}}{n!} \quad (b) \sum_{n=1}^{\infty} \left( \frac{1}{3} + \frac{1}{n} \right)^{2n}.$$

5. (12) Determine whether the following series are *absolutely* convergent, *conditionally* convergent or divergent. Please state which test you are using.

$$(a) \sum_{n=1}^{\infty} (-1)^n \frac{n^2}{\sqrt{n^6 + n}}, \quad (b) \sum_{n=1}^{\infty} (-1)^n \frac{2^n}{4^n + 2}.$$

6. (12) Find the radius of convergence and interval of convergence for

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{n^2 2^n}.$$

7. (10) Find the first 3 non-zero terms in the Taylor series about  $a = 1$  for  $f(x) = 3 - x + 2x^2$ .
8. (10) Find the Maclaurin series (Taylor series about  $a = 0$ ), up to and including  $x^3$  terms, for the function  $f(x) = e^{-x} \sin(2x)$ .
9. (8) Solve for  $x$

$$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^n}{n!} + \cdots = 2.$$