

Math 112 Exam 3, Fall 2019

Read each problem carefully. Show all your work for each problem! Be sure to identify any tests used to determine convergence or divergence.

1. (10) Determine whether the following sequences $\{a_n\}$ are convergent or divergent. Find the limit of any convergent sequences.

$$(a) a_n = \sqrt{\frac{1+4n}{1+n}}, \quad (b) a_n = \left(\frac{9}{10}\right)^{n-\frac{1}{n}}.$$

2. (10) Sum the following series:

$$(a) \sum_{n=1}^{\infty} \frac{2^n + 3^n}{6^n}, \quad (b) \sum_{n=2}^{\infty} \frac{1}{n(n-1)}.$$

3. (16) Determine whether the following series are convergent or divergent. Please state which test you are using.

$$(a) \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^5+2}}, \quad (b) \sum_{n=1}^{\infty} ne^{-1/n^2}.$$

4. (16) Determine whether the following series are convergent or divergent. Please state which test you are using.

$$(a) \sum_{n=2}^{\infty} \frac{1}{\ln(n^2)}, \quad (b) \sum_{n=1}^{\infty} \frac{2^{2n} + 5^n}{2^n + 6^n}.$$

5. (16) Determine whether the following series are *absolutely* convergent, *conditionally* convergent or divergent. Please state which test you are using.

$$(a) \sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{4n^3+1}}, \quad (b) \sum_{n=1}^{\infty} (-1)^n ne^{-n}.$$

6. (a) (6) Write down the first 3 non-zero terms in the Maclaurin series (Taylor series about $a = 0$) for the function $f(x) = 2xe^x$.

(b) (6) Find the first 3 non-zero terms in the Taylor series about $a = 1$ for $f(x) = \frac{1}{x}$.

7. (10) Find the radius of convergence and interval of convergence for

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{n^2 2^n}.$$

8. (10) Determine whether the following series is convergent or divergent. Please state which test you are using.

$$\sum_{n=1}^{\infty} \left(n \sin\left(\frac{2}{n}\right)\right)^n.$$