Math 111 FINAL EXAM, Spring, 2022

Read each problem carefully. Show all your work for each problem! Use only those methods discussed in class. No Calculators!

1. (8) Evaluate the following limits:

$$(a)\ \lim_{x\to 0}\frac{\sin x^2}{\cos 2x-1},\qquad (b)\ \lim_{x\to 0}\left(e^{2x}+x\right)^{\frac{1}{x}}\qquad (c)\ \lim_{t\to -2}\frac{t^3-2t+4}{t^2-t-6}$$

2. (8) Find dy/dx for each of the following:

$$(a)\ y=e^{x+y}-x,\qquad (b)\ y=x\arcsin(2x),\qquad (c)\ y=x^{1+\frac{1}{\ln x}}$$

3. (8) Evaluate the integrals:

$$(a)\ \int_{-1}^{1}\frac{x}{1+x^2}\,dx,\qquad (b)\ \int\frac{3x^2}{\sqrt{5x^3+9}}\,dx,\qquad (c)\ \int 3x^2(1-x^{-6})\,dx$$

4. (a) (8) Find the area of the region bounded by the curves $y=x^2-6$ and $y=2x+2$.

   (b) (14) A rectangular plot of land will be bounded on one side by a stream and the other three sides by a fence. If 800 ft^2 of land is to be enclosed, what are the dimensions that will require the least amount of fencing? Show that your result is a minimum.

5. (8) Find dy/dx for each of the following:

$$(a)\ y=\ln(\cos e^x),\qquad (b)\ y=x\tan(2\sqrt{x})+4,\ \ (c)\ y=\int_{\sqrt{x}}^{0}\cos t^2\,dt$$

6. (a) (4) Find the average value of the function $f(t)=3t^2-2t-4$ over the interval $[1,3]$.

   (b) (10) Air is being pumped into a spherical balloon at a rate of 20 in^3/s. How fast is the radius increasing when the surface area is 4 in^2? (Recall, the volume and surface area of a sphere are given by $V=\frac{4}{3}\pi r^3$ and $A=4\pi r^2$).

7. (16) Consider the function $y=\frac{4+x^2}{x}$.

   (a) Find the intervals on which this function is increasing or decreasing

   (b) Find the intervals on which this function is concave up or concave down

   (c) Find all asymptotes; horizontal, vertical and/or slant.

   (d) Determine the points (if any) at which this function has a local maximum, a local minimum or a point of inflection

   (e) Sketch this function making sure to label the points found in part **d**.