# Math 111 Final EXAM, December 18, 2019

Read each problem carefully. Show all your work for each problem! Use only those methods discussed in class. No Calculators!

1. (8) Evaluate the following limits:

$$(a) \ \lim_{x \to 0} \frac{e^x - 1}{x}, \qquad (b) \ \lim_{x \to \infty} \left( \frac{x}{x+1} \right)^x.$$

2. (8) Evaluate the following limits:

$$(a) \ \lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{\sin x} \right), \qquad (b) \ \lim_{x \to 1^+} (\ln x) \ln(x - 1).$$

3. (8) Find $dy/dx$ for each of the following:

$$(a) \ y = x \cos(x), \qquad (b) \ y = \arctan(xy).$$

4. (10) For the function below, find all of the following if they exist: (i) all local extrema, (ii) points of inflection, (iii) intervals where the function is increasing or decreasing, (iv) intervals of upward and downward concavity, and (v) all asymptotes. Also, sketch a plot of the curve $y = f(x)$.

$$f(x) = \frac{x - 1}{x^2}.$$

5. (8) Evaluate the integrals:

$$(a) \ \int x^2 \sqrt{x^3 + 1} \, dx, \qquad (b) \ \int e^{\tan x} \sec^2 x \, dx.$$

6. (8) Find the area of the region bounded by the curves $y = x^2$ and $y = -3x$.

7. (8) Find $dy/dx$ for each of the following:

$$(a) \ y = \frac{\ln x}{x}, \qquad (b) \ y = \int_{x^2}^{10} \sqrt{1 + t^2} \, dt.$$

8. (8) Find $dy/dx$ for each of the following:

$$(a) \ y = x^{\frac{x}{\ln x}}, \qquad (b) \ y = \sin(\sqrt{x^2 + 1}).$$

9. (8) The area of a square is increasing at a rate of 6 in$^2$/s. At what rate is each side increasing when the length of one side is 3 in?

10. (a) (4) Find $dy/dx$:

$$y = 1 + 2^{\sqrt{x}}.$$

(b) (4) Evaluate the integral

$$\int_{-\pi}^{\pi} \sin x \, \cos x \, dx$$

11. (8) Evaluate the integrals:

$$(a) \ \int \frac{e^x}{e^{2x} + 1} \, dx, \quad (b) \ \int_1^4 \frac{x^{3/2} + 1}{x^{1/2}} \, dx.$$

12. (10) An open-top box with a square base is to be constructed that will hold a volume of 500 cm$^3$. What are the dimensions of this box that will minimize the amount of material to be used? Show that your result is a minimum.