Math 111 Exam 3, Spring 2022

Read each problem carefully. Show all your work for each problem! Use only those methods discussed thus far in class.

1. (14) Evaluate the following limits:

$$(a)\ \lim_{x\to 1}\frac{\ln x}{x^4-x^2}, \qquad (b)\ \lim_{x\to 0}(\cos 2x-\sin 2x)^{1/x}.$$

2. (a) (7) Find the linearization of $f(x)=\ln(1+4x+2x^2)$ about $a=0$.
   (b) (7) Use finite approximation to estimate the area under the graph of $f(x)=4x^2$ between $x=0$ and $x=1$ using a lower sum with two rectangles of equal width.

3. (14) Find the absolute maximum and minimum values of the following function on the given interval:

$$f(x)=x^{3/2}-3\sqrt{x}, \qquad 0\le x\le 4.$$

4. (14) A rectangular plot of land will be bounded on one side by a stream and the other three sides by a fence. With 40ft of fence at your disposal, what is the largest area you can enclose? Show that your result is a maximum.

5. (a) (7) Use Newton's method to estimate a solution of $f(x)=x^4-2x+2=0$. Start with $x_0=0$ and then find x_2.
   (b) (7) Find the most general antiderivative or indefinite integral:

$$\int(1+\sec\theta)\cos\theta\ d\theta$$

6. (14) Evaluate the following limits:

$$(a)\ \lim_{x\to 0}\frac{e^{x^2}-1}{\cos x-1}, \qquad (b)\ \lim_{x\to\infty}(\sqrt{9x^2+12x}-3x)$$

7. (16) Consider the function $y=\frac{x^2+1}{x}$.
   (a) Find the intervals on which this function is increasing or decreasing
   (b) Find the intervals on which this function is concave up or concave down
   (c) Find all asymptotes
   (d) Determine the points (if any) at which this function has a local maximum, a local minimum or a point of inflection
   (e) Sketch this function making sure to label the points found in part **d**.