Math 111 Exam 3, Fall 2019

Read each problem carefully. Show all your work for each problem! Use only those methods discussed thus far in class.

1. (12) Evaluate the following limits:

$$(a)\ \lim_{x\to 1}\frac{\sin(\pi x)}{\ln x}, \qquad (b)\ \lim_{x\to 1} x^{\frac{1}{x-1}}.$$

2. (12) Find the linearization of $f(x) = \ln(2x-1)$ about $a = 1$. Then use this linear approximation to estimate the value of $f(1.2)$.

3. (12) Find the absolute maximum and minimum values of the following function on the given interval:

$$y = 6\sqrt{x} - 2x^{3/2}, \quad 0 \le x \le 4.$$

4. (12) An open-top circular cylindrical tank is to be built that will hold a volume of 1000 π m^3. What are the dimensions of this tank that will minimize the amount of material to be used? Show that your result is a minimum.

5. (12) Use Newton's method to estimate a solution of $f(x) = x^3 - 9x + 9 = 0$. Start with $x_0 = 0$ and then find x_2.

6. (12) The side of a square is increased from 12 in. to 12.2 in. Use differentials to estimate the resulting change in area.

7. (12) Evaluate the following limits:

$$(a)\ \lim_{x\to 0}\frac{\arctan(x) - x}{x^3}, \qquad (b)\ \lim_{x\to \frac{\pi}{2}} (\sec x - \tan x)$$

8. (16) Consider the function $y = 3x^4 - 4x^3$.

   (a) Find the intervals on which this function is increasing or decreasing

   (b) Find the intervals on which this function is concave up or concave down

   (c) Determine the points (if any) at which this function has a local maximum, a local minimum or a point of inflection

   (d) Sketch this function making sure to label the points found in part **c**.