Math 111 EXAM II, October 23, 2019

Read each problem carefully. Show all your work for each problem! No Calculators! For tangent lines, both point-slope and slope-intercept forms are acceptable.

1. (12) Find dy/dx for the following:

$$(a) \ \ y = \frac{2^x}{x^2}, \qquad (b) \ \ y = e^{\sqrt{\pi + \sqrt{x+1}}}.$$

2. (8) Find $\frac{d^3y}{dx^3}$ (i.e. the third derivative of y) for $y = \log_3 x$.

3. (12) Find dy/dx for the following:

$$(a) \ \ e = x^y, \qquad (b) \ \ (x^2 + y^2)^2 = 4x^2 y.$$

4. (10) At time $t \geq 0$, the position of a body moving along the s-axis is $s = 4 \arctan(t/2) - t$. Determine the time when the body changes direction. What are the body's position and acceleration at this time?

5. (a) (6) Find $y'(x)$ and simplify:

$$y = \sin^{-1}(e^{-x}) + \sec^{-1}(e^x), \qquad x \geq 0.$$

   (b) (6) Find the equation of the tangent line to $y = x^{(1+1/\ln x)}$ at $x = e$.

6. (12) Use the definition of the derivative as the limit of a difference quotient to find the derivative of $f(x) = \frac{1}{x} + 1$.

7. (10) Suppose that the differentiable function $y = f(x)$ has an inverse and that the graph of f passes through the point $(1, 3)$ and has a slope of $1/2$ there. Find the equation of the line tangent to the function $f^{-1}(x)$ at $x = 3$.

8. (12) Find the equation of the tangent line to $y = \sin(\arccos 2x)$ at $x = 1/4$.

9. (12) The volume of an expanding sphere is increasing at a rate of 16π ft^3/s when $r = 1$ft. How fast is the surface area increasing at that instant? (Recall: $V = \frac{4}{3}\pi r^3, \quad A = 4\pi r^2$).