Math 111 EXAM I, Spring 2022

Read each problem carefully. Show all your work for each problem! Use only those methods discussed thus far in class. When evaluating limits, you must use algebraic methods and/or the basic trig limit, and allow $+\infty$ and $-\infty$ as possible values of a limit. No Calculators!

1. (14) Evaluate the following limits:

$$(a)\ \lim_{x\to 0}\frac{\tan 3x^2}{x\sin x}, \qquad (b)\ \lim_{x\to+\infty}\frac{\sqrt{4x^2+2x}+4x}{2x-1}.$$

2. (14) Evaluate the following limits:

$$(a)\ \lim_{x\to 0}\left(\frac{\sin 4x}{2x}-\frac{2x}{\sin 4x}\right), \qquad (b)\ \lim_{x\to 1^-}\frac{2+x}{x^2-3x+2}.$$

3. (14) Evaluate the following limits:

$$(a)\ \lim_{t\to 3}\frac{t^2+2t-15}{t^2-2t-3}, \qquad (b)\ \lim_{x\to 2}\frac{x-2}{\sqrt{x+7}-3}.$$

4. (a) Find the average rate of change of $f(x)=\sqrt{9+5x^3}$ over the interval $-1\le x\le 2$,
   (b) Use the definition of the derivative to find the derivative of the function $f(x)=1/x$.

5. (14) Find the constants a and b so that the function given below is continuous for all x:

$$f(x)=\begin{cases} a\,\dfrac{\sin 2x}{x}, & x<0 \\ x+b, & 0\le x\le 1 \\ \dfrac{x^2-1}{x-1}, & x>1. \end{cases}$$

6. (14) Find an equation for all asymptotes, if they exist, for the following function, and be sure to label the type of asymptote (horizontal, vertical or oblique (slant)):

$$y=\frac{6x^2+8x-5}{2x+4}.$$

7. (14) Find all points where the following function is discontinuous and identify the type of discontinuity (jump, infinite or removable). If the function has a removable discontinuity, then determine how to define f at that point in a way that extends $f(x)$ to be continuous there.

$$y=\frac{x^2+2x}{x^2+6x+8}.$$