Math 111 EXAM I, September 25, 2019

Read each problem carefully. Show all your work for each problem! Use only those methods discussed thus far in class. When evaluating limits, you must use algebraic methods and/or the basic trig limit, and allow $+\infty$ and $-\infty$ as possible values of a limit. No Calculators!

1. (12) Evaluate the following limits:
$$(a)\ \lim_{x\to 2}\sqrt{\frac{2x^2+1}{3x+3}}, \qquad (b)\ \lim_{\theta\to 0}\frac{\tan(\pi\theta)}{\theta}.$$
2. (8) Evaluate the following limit:
$$\lim_{x\to 1^-}\frac{|2-x|}{x^2-3x+2}.$$
3. (8) Evaluate the following limit:
$$\lim_{t\to -3}\frac{t^2+4t+3}{t^2-9}.$$
4. (10) Evaluate the following limit:
$$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}.$$
5. (10) Evaluate the following limit:
$$\lim_{x\to +\infty}\frac{x\sqrt{x^2+4}}{2x^2+3x-6}.$$
6. (12) Consider the function $f(x)=\frac{2}{x}$:
   (a) Find the average rate of change of f over the interval $1\le x\le 2$,
   (b) Use the definition of the derivative to find the slope of the tangent line at $x=2$, and then find the equation of the tangent line to the curve at this point.
7. (12) Find the constants a and b so that the function given below is continuous for all x:
$$f(x)=\begin{cases} x^2+3, & x<2\\ a, & x=2\\ ax+b, & x>2.\end{cases}$$
8. (12) Find all horizontal, vertical and oblique (slant) asymptotes, if they exist, for the following function:
$$y=\frac{x^2-2x-8}{x+1}.$$
9. (8) Find all points where the following function is discontinuous and identify the type of discontinuity:
$$y=\frac{x|x-1|}{(x^2-x)(x-2)}.$$
10. (8) Evaluate the following limit (hint: begin by multiplying numerator and denominator by the conjugate):
$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\sin(x)}.$$