Math 213 **Exam 1** September 27, 2006

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

1. (18 points) For the curve defined by

$$\mathbf{r}(t) = (1+t^2)\mathbf{i} + (-3+2t)\mathbf{j} + t^3\mathbf{k},$$

   find:

   (a) the derivative $\mathbf{r}'(t)$ and the unit tangent vector $\mathbf{T}(t)$;
   (b) an equation of the tangent line through point $(2,-1,1)$;
   (c) the curvature κ at point $(2,-1,1)$.

2. (16 points) Reduce the equation

$$x^2 - 2x + y^2 + 4z^2 + 2y - 16z + 14 = 0$$

   to the standard form. Then classify this surface and sketch it.

3. (16 points) Compute (in (a), $\mathbf{a}$ and $\mathbf{b}$ are constant vectors):

   (a) $\dfrac{d}{dt}\left[(\mathbf{a}+t\mathbf{b})\times(\mathbf{b}-t\mathbf{a})\right]$.
   (b) $\displaystyle\int_0^{\pi}\left(e^{-t}\,\mathbf{i} - \sqrt{t}\,\mathbf{j} + \cos t\,\mathbf{k}\right)dt$.

4. (16 points) Given the two planes

$$2x - z = 3, \qquad 2x + 3y + z = 0,$$

   find:

   (a) the cosine of the angle between them;
   (b) an equation for the line of their intersection.

5. (16 points)

   (a) Plot the point P whose cylindrical coordinates are $(\sqrt{2}, \pi/4, \sqrt{6})$ and then find its spherical coordinates.
   (b) Describe in words the surface whose equation in cylindrical coordinates is $\theta = \frac{2\pi}{3}$.

6. (18 points)

   (a) Find a vector and a parametric equation of the line passing through points $P(1,2,3)$ and $Q(2,1,5)$.
   (b) Reparametrize the curve

$$\mathbf{r}(t) = 4t\,\mathbf{i} - 3\sin t\,\mathbf{j} + 3\cos t\,\mathbf{k}$$

   with respect to the arclength measured from the point where $t=0$ in the direction of increasing t.