Glycerol & Nematic Liquid Crystal in a Hele-Shaw Cell: Electric Field Effects

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Abstract

A Hele-Shaw cell consists of two transparent plates (glass or plexiglass), narrowly spaced, with a possible hole in one plate to inject air or fluid. As part of a senior year Capstone course in Applied Mathematics (Math 451H) we carried out a series of experiments in order to investigate the stability of free interfaces between fluids, specifically glycerol/air, glycerol/water, and nematic liquid crystal/air interfaces.

For glycerol, we carried out two sets of experiments. Our first set consisted of (i) pressing a circular blob of glycerol, surrounded by air, between two glass plates with foam rubber spacers ($stable$ in the pressing phase; $unstable$ when the plates spring apart again); and (ii) pressing the plates together when a circular air bubble is trapped, surrounded by glycerol ($unstable$ in the pressing phase; $stable$ when the plates spring apart). In these experiments there was no hole in either plate. In our second set of experiments an air (or water) bubble was injected into a blob of glycerol trapped between the plates, to recreate the observed ($unstable$) radial fingering described by Paterson [1].

For Nematic Liquid Crystal (NLCs), we repeated the air bubble injection experiment, but also applied an electric field across the plates. This alters the effective viscosity of the NLC. We were able to verify, qualitatively, the results found by Tóth-Katona & Buka [2].

Linear Stability Results

For all cases of interest an exact solution for a Newtonian fluid in a Hele-Shaw cell is an expanding or contracting circular interface between the two fluids. We consider the air/glycerol interface for simplicity, assuming that the air is at uniform pressure. If a perturbation of wavenumber n is imposed on the interface (representing n fingers beginning to form) we may carry out linear stability analysis of the nearly-circular interface (see, e.g. [1]). This leads to an instability criterion, from which the fastest-growing wavelength (or wavenumber, n) is determined by differentiating the growth-rate λ (where positive) with respect to n. This leads to the following formulae for the expected number of fingers, which we apply to analyze both the glycerol and nematic liquid crystal experiments. For reasons of space we present only selected results here.

Glycerol Blob Between Plates

$$n = \sqrt{\frac{1}{3} + \frac{2b' R_0^3}{b^3 \sigma}} \tag{1}$$

where, n is the number of fingers, R_0 is the inital bubble radius, $b = b(t)$ is the plate separation, $b' = \frac{db}{dt}$ is the velocity of the top plate and σ is the surface tension at the air/glycerol interface.

Air Bubble Injected into Glycerol

$$n = \frac{1}{\sqrt{3}} \sqrt{\frac{6\mu Q^{\frac{3}{2}} t^{\frac{1}{2}}}{\pi^{\frac{3}{2}} b^2 \sigma} + 1} \tag{2}$$

where μ is the viscosity of glycerol, Q is the rate of change of bubble area, t is the time after injection, and other variables are as above.

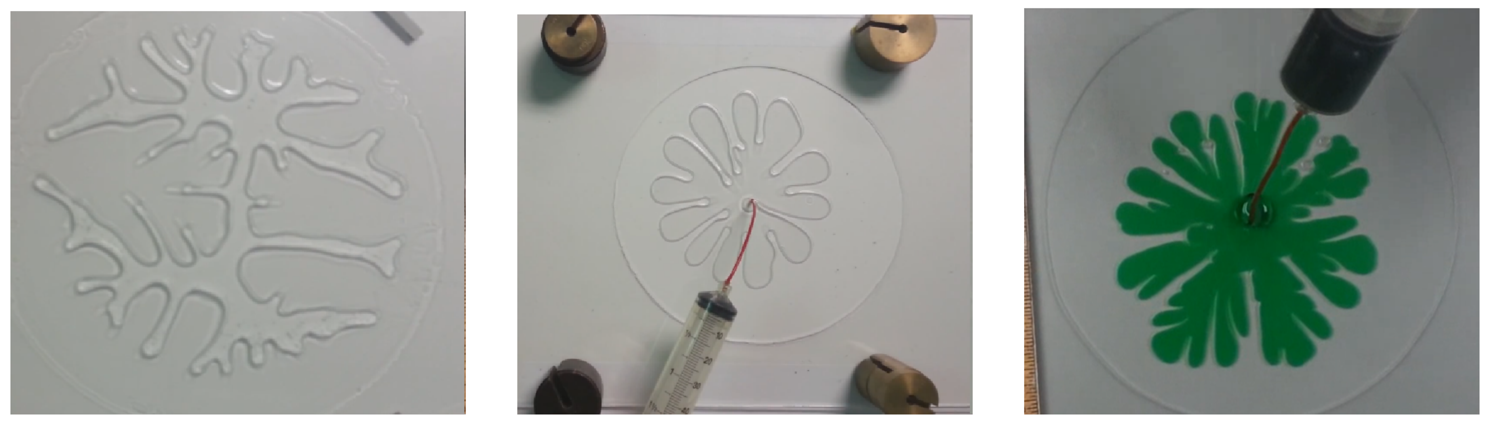
Glycerol Experimental Results

Blob of Glycerol Pressed Between Two Plates

As described above, pieces of foam rubber separated the plates so as to allow the top plate to spring back up once the force on the top plate was released. We describe a typical experiment here, shown in Fig. 1(a). Relevant material parameters are $\mu_{\text{glycerol}} = 1.412$ Pa.s, $\sigma = 0.064$ Nm^{-1}.

At maximum compression (minimum plate separation) the blob radius was 0.35 m, giving blob area 9.3×10^{-2} m^2. The space between the plates once pressed down was 3.5×10^{-3} m. The plates took 72 s to fully spring back, at which point the plates were 4×10^{-3} m apart.

Glycerol Experimental Results (continued)



(a) Pressing the plates together (b) Injection of air bubble (c) Injection of water

Figure 1: Selected experimental images

Inward fingering of air into the glycerol was observed during the spring-back phase. We counted a range of 'fingers' increasing from 8 to 35 over the full course of the spring-back phase.

Evaluating Eq.(1) we obtain $n \approx 19$ for the predicted number of fingers. This is reasonable considering the following points: we assumed a constant injection rate in deriving Eq.(1); we treated the air phase as passive; and (as is well-known) the usual Hele-Shaw dynamic boundary condition "$p = \sigma\kappa$" (pressure equals surface tension times curvature) is only an approximation of the true boundary condition.

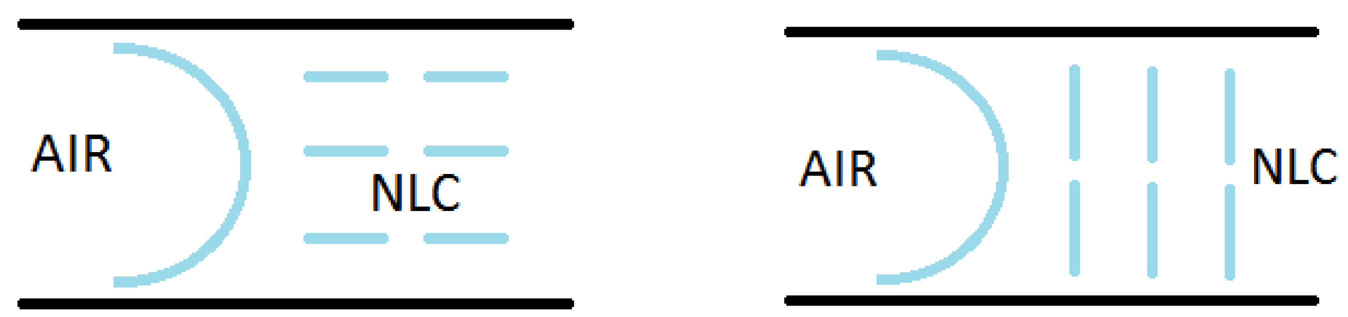
Air Bubble Injected Into Glycerol

This experimental setup was very similar; however here we drilled a hole in the top plate to allow injection of air into the glycerol. We initially covered the hole with tape to prevent glycerol oozing out through the hole when initially assembling the cell. The spacing between plates was fixed at 1.31×10^{-3} m for this experiment. We measured the radius and area of the glycerol blob before air injection to be 2.15×10^{-1} m, and 3.02926×10^{-2} m^2, respectively. We then injected 2×10^{-5} m^3 of air into the hole using a syringe, over a time of 7.15 s.

The interface exhibited unstable fingering, with a total of 12 outward fingers forming. Using the measurements recorded above we found the average rate of change of the air bubble area to be $Q = 9.78881 \times 10^{-4}$ m^{2}s^{-1}. The prediction of Eq.(2) gives $n \approx 17$ as the expected number of fingers, again very reasonable agreement given the limitations of both model and experimental setup.

Liquid Crystal Experimental Results

Nematic Liquid Crystals differ from Newtonian fluids (such as glycerol) in several ways. They typically consist of long, rod-like molecules, which have a tendency to align with each other. They also have a preferred orientation at a substrate, a phenomenon known as surface $anchoring$; and the molecules will align with an applied electric field of sufficient strength. Our experimental setup uses all these features. We take the Hele-Shaw setup used in the air injection experiment above, and coat the interior of the plates with Indium Tin Oxide (ITO), providing a transparent conducting layer. We then coat the ITO layer with a thin film of PolyVinyl Alcohol (PVA), which is rubbed with a soft cloth after drying. This PVA treatment imparts $planar\ anchoring$ to the plates of the cell: the NLC molecules prefer to lie parallel to the plates in the absence of other forces. With a sufficiently strong field applied across the plates however, this anchoring may be broken and the molecules forced to stand perpendicular to the plates. See Figure 2.



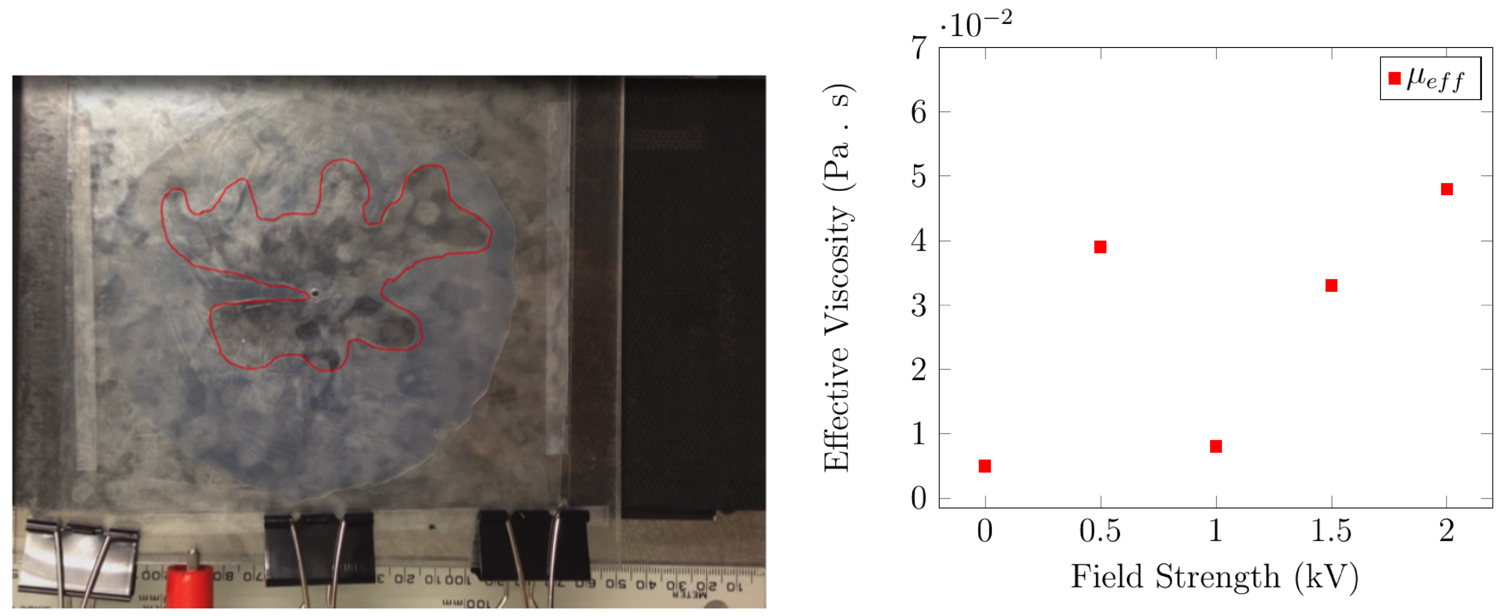
(a) Anchoring dominated: Low flow resistance (b) Field dominated: High flow resistance

Figure 2: Field off and field on states in the NLC experiments

Liquid Crystal Experiments (continued)

We carried out a series of air injection experiments at different applied electric fields ranging from 0 to 2000 V, for a Hele-Shaw cell of constant thickness 0.0043 m filled with the NLC 5CB (4-Cyano-4'-pentylbiphenyl). For each experiment, we counted the number of fingers that formed and estimated (from our videos of the experiments) the air injection rate Q. Assuming the surface tension coefficient for an air-water interface, we then used Eq.(2) for each experiment to calculate the viscosity μ_{eff} corresponding to the observed number of fingers.

Figure 3(a) shows a typical experimental image. The air/NLC interface is highlighted in red: our treatment of the plates with PVA to achieve the desired anchoring made them rather opaque. Possible nonuniformities in the planar anchoring, which may lead to preferred spreading directions, also led to bubble shapes that were less symmetric than for the isotropic glycerol experiments. Figure 3(b) shows the plot of effective viscosity μ_{eff} versus applied field that we obtained.



(a) Sample experimental result at field strength 2kV (b) Effective viscosity vs electric field strength

Figure 3: An experimental image from our trials, and the final plot of effective viscosity versus applied field strength for all our trials

As the voltage was increased, the calculated viscosity also increased. The data point for 500 V has an anomalously high value, attributable to human and/or experimental error; with more trials, we would expect to see a monotone, though not necessarily linear, relation between viscosity and voltage.

This trend of increased viscosity with increased E-field may be understood by reference to Fig.2: in the "field off" (or low field) state (Fig.2(a)) the anchoring dominates, while in the high-field case (Fig.2(b)) the field dominates. In the latter case the molecular configuration provides more resistance to flow, and the NLC is more difficult to displace by the air bubble (thus effectively more viscous).

Conclusions

We carried out a series of experiments on glycerol and NLC in a Hele-Shaw cell and compared our findings with simple linear stability analyses. We were able to reproduce, qualitatively, the findings of Paterson [1] and Tóth-Katona & Buka [2]. Our analysis for the NLC experiments is simplistic in the extreme, taking account of the physics of liquid crystals only in a very crude sense; nonetheless our findings are broadly in line with expectations. The reader is referred to [1, 2] for more complete treatments.

References

References

[1] Lincoln Paterson. Radial fingering in a hele-shaw cell. *J. Fluid Mech.*, 113:513–529, 11 1981.

[2] Tibor Tóth-Katona and Ágnes Buka. Nematic liquid crystal/air interface in a radial hele-shaw cell: Electric field effects. *Phys. Rev. E*, 67:041717, Apr 2003.

Acknowledgement

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