Math 112 – Spring 2011
Final Examination

Please complete the following problems. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not allowed during this examination.

1.(16 pts.) Find the volume of the solid obtained by revolving the region enclosed by $y = xe^x, y = 0$ and $x = 1$ about the x-axis.

2.(18 pts.) Evaluate the following integrals:

a. $\displaystyle\int \frac{6x+8}{x(x+2)^2}\,dx$ **b.** $\displaystyle\int x\sec^2(x)\,dx$

3.(18 pts.) Evaluate the following integrals:

a. $\displaystyle\int \frac{x^2+2x+3}{x(x^2+1)}\,dx$ **b.** $\displaystyle\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

4.(18 pts.) Evaluate the following integrals:

a. $\displaystyle\int \sin^3(x)\cos^2(x)\,dx$ **b.** $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

5.(18 pts.) Evaluate the following improper integrals if they are convergent or show they are divergent:

a. $\displaystyle\int_0^\infty \frac{\arctan(x)}{1+x^2}\,dx$ **b.** $\displaystyle\int_0^2 x^2\ln(x)\,dx$

6.(32 pts.) Determine whether the following series are convergent or divergent. If you use a convergence or divergence test, please state which test you are using.

a. $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{(2n)!}$ **b.** $\displaystyle\sum_{n=1}^{\infty}2^{\frac{1}{n}}$

c. $\displaystyle\sum_{n=1}^{\infty}\frac{1+3^n}{1+4^n}$ **d.** $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{n-1}}{3n^2+4}$

7.(16 pts.) Find the radius of convergence and interval of convergence for $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(x-2)^n}{\sqrt{n+1}}$.

8.(12 pts.) Find the first three nonzero terms of the Taylor series about $a = \frac{\pi}{4}$ for the function $f(x) = \cos(x)$.

There are more problems on the next page!

9.(20 pts.) Consider the function $f(x) = xe^{-x^5}$.

a. Find the Maclaurin series (Taylor series about $a = 0$) for $f(x)$.

b. Use the series from part **a** to find a series for $\displaystyle\int_0^{0.1} xe^{-x^5}\,dx$.

c. Approximate the value of the integral from part **b** with an error of absolute value less than 10^{-8}.

10.(16 pts.) Find the area inside the polar curve $r = 2\cos(\theta)$ and outside the polar curve $r = 1$.

11.(16 pts.) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for the curve $x(t) = \dfrac{t}{1+t}$, $y(t) = \ln(1+t)$.