**Math 112 – Fall 2011**
**Final Examination**

Please complete the following problems. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not allowed during this examination.

**1.**(22 pts.) Consider the parametric curve that is given by $x(t) = \cos^2(t),\ y(t) = \sin^2(t)$ for $0 \le t \le \frac{\pi}{4}$.

**a.** Find the length of this curve.

**b.** Find the area of the surface generated by revolving this curve about the $x$-axis.

**2.**(22 pts.) Consider the polar curve given by $r = 4\sin(\theta)$.

**a.** Find the slope of the tangent line to this curve for $\theta = \frac{\pi}{3}$.

**b.** Find the area enclosed by this curve.

**3.**(18 pts.) Evaluate the following integrals:

**a.** $\displaystyle\int \frac{4x+1}{x(x+1)^2}\,dx$ **b.** $\displaystyle\int \frac{1}{(4-x^2)^{\frac{3}{2}}}\,dx$

**4.**(18 pts.) Evaluate the following integrals:

**a.** $\displaystyle\int \frac{3x-1}{x^3+x}\,dx$ **b.** $\displaystyle\int \frac{\ln(x)}{\sqrt{x}}\,dx$

**5.**(18 pts.) Evaluate the following improper integrals if they are convergent or show they are divergent:

**a.** $\displaystyle\int_0^1 \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$ **b.** $\displaystyle\int_0^\infty xe^{-x}\,dx$

**6.**(16 pts.) Find the volume of the solid obtained by revolving the region enclosed by $y = \dfrac{1}{x^2+1}$, $y = 0$, $x = 0$ and $x = 1$ about the $x$-axis.

**7.**(26 pts.) Determine whether the following series are convergent or divergent. If you use a convergence or divergence test, please state which test you are using.

**a.** $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^4+4}}$ **b.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^n+1}$

**c.** $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+5^n}{4^n+5^n}$

**8.**(16 pts.) Find the radius of convergence and interval of convergence for $\sum_{n=1}^{\infty} \frac{3^n(x-1)^n}{n}$.

**9.**(22 pts.) Consider the function $f(x) = x\cos(x^2)$.

**a.** Find the Maclaurin series (Taylor series about $a = 0$) for $f(x)$.

**b.** Use the series from part **a** to evaluate the following: $\lim_{x\to 0} \frac{x\cos(x^2) - x}{3x^5}$.

**10.**(22 pts.) Consider the function $f(x) = e^x$.

**a.** Find the first four nonzero terms of the Taylor series about $a = 2$ for $f(x)$.

**b.** Suppose $f(x)$ is approximated by the Taylor polynomial from part **a**. Estimate the error when $1 \leq x \leq 3$.