Math 112 – Spring 2011
Examination 3

Please complete the following problems. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not allowed during this examination.

1.(12 pts.) Determine whether the following series are convergent or divergent. If you use a convergence or divergence test, please state which test you are using.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{2^n+1}$ **b.** $\displaystyle\sum_{n=1}^{\infty} \frac{(1+\ln(n))^n}{n^n}$

2.(12 pts.) Determine whether the following series are convergent or divergent. If you use a convergence or divergence test, please state which test you are using.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{1+3e^n}$ **b.** $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^{\frac{3}{2}}}$

3.(16 pts.) Determine whether the following series are *absolutely* convergent, *conditionally* convergent, or divergent. If you use a convergence or divergence test, please state which test you are using.

a. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\arctan(n)}{n^2}$ **b.** $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{1+\sqrt{n}}$

4.(12 pts.) Determine whether the following series are convergent or divergent. If you use a convergence or divergence test, please state which test you are using.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^2 10^n}$ **b.** $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{2n^5-1}}$

5.(12 pts.) Find the radius of convergence and interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{10^n(x-1)^n}{n!}$.

6.(8 pts.) Find the first three nonzero terms of the Maclaurin series (Taylor series about $a=0$) for $f(x) = x^2\sin(2x)$.

7.(12 pts.) Find the radius of convergence and interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{n\,4^n}$.

8.(16 pts.) Consider the function $f(x) = e^x$.

a. Find the first four nonzero terms of the Taylor series about $a = 1$ for $f(x)$.

b. Suppose $f(x)$ is replaced by the Taylor polynomial from part **a**. Estimate the error when $0 \le x \le 2$.