Math 112 – Fall 2011 Examination 2

Please complete the following problems. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not allowed during this examination.

1.(12 pts.) Determine whether the following sequences $\{a_n\}$ are convergent or divergent. Find the limit of any convergent sequences.

$$\mathbf{a.} \ a_n = \frac{\ln(n)}{\sqrt{n}}$$

b.
$$a_n = \ln(2n) - \ln(n+1)$$

2.(18 pts.) Consider the integral
$$\int_1^5 \frac{1}{x^2 + x} dx$$
.

- **a.** Evaluate this integral.
- **b.** Estimate this integral using the trapezoid rule with n=4 steps.
- **c.** Estimate this integral using Simpson's rule with n=4 steps.
- **3.**(14 pts.) Evaluate the following integrals:

a.
$$\int \frac{1}{\sqrt{1+9x^2}} \, dx$$

b.
$$\int \frac{x^2}{(x-1)^3} dx$$

4.(14 pts.) Evaluate the following integrals:

a.
$$\int x \arctan(x) dx$$

b.
$$\int \frac{x^2}{(4-x^2)^{\frac{3}{2}}} \, dx$$

5.(14 pts.) Evaluate the following integrals:

$$\mathbf{a.} \int \frac{4x - 8}{x^3 + 4x} \, dx$$

b.
$$\int x \cosh(x) \, dx$$

6.(14 pts.) Evaluate the following integrals if they are convergent or show they are divergent:

$$\mathbf{a.} \int_0^\infty x e^{-x^2} \, dx$$

b.
$$\int_0^{\frac{\pi}{2}} \tan(x) \sec^2(x) dx$$

7.(14 pts.) Determine whether the following integrals are convergent or divergent. If you use a convergence or divergence test, please state which test you are using.

$$\mathbf{a.} \int_{1}^{\infty} \frac{e^{\frac{1}{x}}}{x^3} dx$$

b.
$$\int_{1}^{\infty} \frac{x}{\sqrt{x^3 + 2}} dx$$