MATH 337-FINAL EXAM May 12, 2010

1.(15 points) Let $v_1 = (1, -1, 5)^T$, $v_2 = (-4, 2, -6)^T$, $v_3 = (9, -4, 10)^T$, $v_4 = (-7, 1, 7)^T$ be the columns of the matrix A.

a) Find bases for Nul(A), Col(A), and Row(A).

b) Find the rankA and the orthogonal complement $V^{\perp}$ of $V = col(A)$.

2. (20 points) a) Let $v_1 = (4, 2, 0)^T$, $v_2 = (0, 5, 0)^T$, and $v_3 = (-2, 4, 5)^T$ be the columns of the matrix A. Find the eigenvalues of A.

b) Find bases for the corresponding eigenspaces.

c) Diagonalize A (i.e. write it as $A = PDP^{-1}$). Do not compute P^{-1}.

d) Using part c), find det(A).

3. (15 points) Let $A = [v_1 v_2 v_3]$ with $v_1 = (1, 1, 0)^T$, $v_2 = (1, 2, 0)^T$, and $v_3 = (0, 1, 2)^T$.

a) Use the Gram-Schmidt method to find an orthogonal basis for V=Col(A).

b) Find the QR factorization of A.

4. (20 points) a) Write down the matrix A corresponding to the quadratic form $Q(x) = 2x_1^2 + 10x_1x_2 + 2x_2^2$.

b) Is Q positive definite, negative definite, or indefinite?

c) Othogonally diagonalize A and find A^{2010}.

d) (5 pts extra credit) Find the change of variable $x = Py$ that transforms Q into a quadratic form with just square terms. Find this quadratic form.

5. (15 points) Let $u = (3, 6, 0)^T$, $v = (1, 2, 2)^T$ and $V = span\{v\}$.

a) Compute $w = \text{proj}_V u$.

b) Write u as the sum of w and a vector orthogonal to v.

c) Find the distance from u to V.

6. (15 points) Let $A = [v_1 v_2 v_3]$ with $v_1 = (5, 0, 0)^T$, $v_2 = (7, 2, -6)^T$, and $v_3 = (9, 4, -8)^T$.

a) Are the columns of A linearly independent?

b) Is A invertible (no computation is necessary)?

c) Is the equation $Ax = b$ solvable for each b in R^3? Find its solution(s) when solvable. Justify your answer.