MATH 337 – FINAL EXAM – FALL 2011

Show all work and justify all steps of each argument you make.

1)(20 points) Let $A = [v_1 v_2 v_3 v_4]$ with $v_1 = (1,2,3)^T$, $v_2 = (2,4,6)^T$, $v_3 = (3,8,7)^T$, $v_4 = (5,12,13)^T$, and $b = (b_1, b_2, b_3)^T$. Is the system $Ax = b$ consistent for all b in R^3?

b) Find the general solution in the form $x = x_h + p$ of $Ax = (0,6,-6)^T$

c) What is the definition of a basis of a vecror space ? Find bases and dimensions of Nul(A), Col(A) and Row(A). What is the rank of A?

d)(5 points extra credit) Find a basis for $(Row(A))^{\perp}$.

2) (15 points) Let $u = (1,1,0)^T$, $v = (0,1,1)^T$ and $V = span\{v\}$.

a)Find the projection $w = \text{proj}_V u$ of u onto V and its lenght $\|w\|$.

b) Find $\|u - w\|$ and the distance from u to V.

c) What is the angle between u and v?

3) (25 points) Let $A = [x_1 x_2 x_3]$ with $x_1 = (4,1,-2)^T$, $x_2 = (0,3,2)^T$ and $x_3 = (0,0,4)^T$.

a) Find the characteristic equation and the eigenvalues of A.

b) Find bases for the corresponding eigenspaces.

c) Diagonalize A (i.e., write it as $A = PDP^{-1}$). Do not compute P^{-1}.

d) Show that detA=detD and find detA^2A^T. Is A invertible? Explain.

4) (20) Let $A = [v_1 v_2 v_3]$ with $v_1 = (1,-1,0)^T$, $v_2 = (2,0,-2)^T$ and $v_3 = (3,-3,3)^T$.

a) Show that $\{v_1, v_2, v_3\}$ is a basis for Col(A).

b) Find an orthogonal basis for $V = Col(A)$ (Gram-Schmidt)

c) Find the QR factorization of A. Express R in terms of Q and A, but don't compute it.

5) (20 points) a) Find the matrix A corresponding to the quadratic form $Q(x) = 3x_1^2 + 10x_1x_2 + 3x_2^2$.

b) Is Q positive definite, negative definite, or indefinite?

c) Orthogonally diagonalize A. Find A^k where k is a positive integer.