Math 222 FINAL EXAM, December 14, 2007

Read each problem carefully. Show all your work for each problem. No Calculators!

1. (a) (10) Solve the initial value problem $X' = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} X,\ X(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$

   (b) (10) Find all eigenvalues and eigenfunctions of the boundary value problem: $y'' + \lambda y = 0$, $y'(0) = 0,\ y'(\text{L}) = 0$

2. (a) (6) Sketch the odd periodic extension of period 2 of $f(x)$:

$$f(x) = \begin{cases} 0, & 0 \leq x < 1/2 \\ 1, & 1/2 \leq x < 1 \end{cases}$$

(Sketch over the interval $[-3, 3]$)

   (b) (10) Find the Fourier Sine series of $f(x) = 1 - x,\ 0 \leq x < 1,$ with period 2.

3. Find the inverse Laplace Transform of the following functions:

   (a) (8) $F(s) = \frac{2s+1}{s^2+2s+5}$

   (b) (8) $G(s) = \frac{(1-s)e^{-2s}}{(s+1)^3}$

4. (a) (8) Find the solution $y(t)$ of the initial value problem
   $y'' + y = \delta(t - 2\pi) + \alpha u_{3\pi}(t),\quad y(0) = 0,\ y'(0) = 0$

   (b) (4) Calculate $y(\frac{5\pi}{2})$

   (c) (4) Find the value of α such that $y(\frac{7\pi}{2}) = 0.$

5. State if the following IVP's are linear or nonlinear, and solve the problems:

   (a) (8) $ty' + 2y = \frac{cos(t)}{t},\ t > 0,\ y(\pi) = 0$

   (b) (8) $y' = \frac{2x}{y+x^2y}$, $y(0) = -2$

6. (a) (8) Determine the appropriate form of a particular solution $\boldsymbol{y_p}$ for the differential equation: $y'' + 6y' + 13y = e^{-3x}sin(2x) + x^2cos(3x).$
   (**DO NOT** evaluate the constants in $\boldsymbol{y_p}$.)

   (b) (8) Given that $y_1 = x^2$ is a solution of the differential equation
   $x^2\,y'' - 3xy' + 4y = 0,\ x > 0,$
   use reduction of order to find the second linearly independent solution.