Math 222 EXAM III, November 28, 2007

Read each problem carefully. Show all your work for each problem. No Calculators!

1. A mass weighing 8 lb is attached to a spring and to a viscous damper. The damping constant is 2 lb-sec/ft and the spring constant is $\frac{25}{4}$ lb/ft. The mass is set in motion from its equilibrium position with an initial velocity of 4 ft/sec in the downward direction.
   (a) (10) Determine the position of the mass at any time t.
   (b) (6) Find the time t_1 when the mass **first** returns to its equilibrium position.

2. (a) (10) Use the definition of Laplace Transform to find the Laplace Transform of
$$f(t) = \begin{cases} e^t, 0 \le t < 1 \\ 2, \ 1 \le t < \infty \end{cases}$$
   (b) (10) Find the Laplace Transform of $g(t) = \int_0^t e^{-\tau} sin(t-\tau) d\tau$

3. Find the inverse Laplace Transforms of $F(s)$ and $G(s)$:
   (a) (8) $F(s) = \frac{1}{(s+1)(s^2-4)}$
   (b) (8) $G(s) = \frac{s+3}{s^2+4s+8} e^{-4s}$

4. Solve the following IVP's:
   (a) (8) $y'' - y = \delta(t-1), \quad y(0) = 1, \ y'(0) = 0$
   (b) (8) $y'' + y = 2u_1(t) - 4u_3(t), \quad y(0) = 0, \ y'(0) = 0$

5. (a) (10) Assume G(s)=$\mathcal{L}$[g(t)] and use a convolution integral to solve the IVP:
$y'' + 4y = g(t), \quad y(0) = 0, \ y'(0) = 0$
   (b) (6) If $g(t) = 4 \ cos(\omega t)$, determine the frequency ω for which the system in problem 5(a) will exhibit resonance.

6. (a) (10) Transform the given ODE into a system of first order ODE's:
$y^{(4)} = (t+1)y^{(3)} - ty^{(2)} + 2y' - 4y + 5t$
   (b) (6) Verify whether $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$ is (or is not) a solution of the system
$$x' = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} x + \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{2t}.$$