Math 222 EXAM II,   October 24, 2007

Read each problem carefully. Show all your work for each problem. No Calculators!

1. (a)   (8) Determine if the functions y_1, y_2 are linearly dependent or independent:

$$\text{(i)} \quad y_1 = |t-1|, \quad y_2 = 2(t-1), \qquad \text{(ii)} \quad y_1 = 3t+1, \quad y_2 = t+3$$

(b)   (8) Find a function $g(x)$ which satisfies the conditions: $W(f,g) = x$, $f(x) = x$.

2. (a)   (12) Use the method of undetermined coefficients to find a particular solution of the differential   equation

$$y'' - y' = 2e^t - 1 - t$$

(b)   (6) Determine the general solution of the above equation

3. (a)   (12) Given that $y_1 = e^{-x}$ is a solution of the differential equation

$$xy'' + (x-1)y' - y = 0, \quad x > 0,$$

use the method of reduction of order to find the second linerly independent solution y_2.

(b)   (6) Determine the homogeneous ODE whose general solution is

$$y = c_1 e^t + c_2 t e^t + e^{-t}(c_3 \cos 2t + c_4 \sin 2t)$$

4. (16)   Use the method of variation of parameter to find a particular solution of the differential equation

$$2y'' + 4y' + 2y = \tfrac{1}{t}e^{-t}, \ t > 0$$

5. (16)   Determine the form of particular solution of the following ODE, using the method of undetermined coefficients. Do NOT evaluate the constants.

$$y^{(4)} + 2y^{(3)} + 2y'' = 4e^t - 2e^{-t}\cos(t) + te^{-t}$$

6. (16)   Solve the initial value problem

$$y^{(3)} - y'' - y' + y = 0, \quad y(0) = 2, \ y'(0) = -1, \ y''(0) = 0$$