

**Math 222 FINAL EXAM May 4, 2012**

Read each problem carefully. Show all your work. No calculators!

1. Find the general solution of each of the following differential equations:
  - (a) (4)  $\frac{2}{x} \frac{dy}{dx} = y - 4$
  - (b) (4)  $y'' - 6y' + 9y = 0$
  - (c) (8)  $y'' + 3y' + 2y = 4x + 2$
  
2. (14) Solve the IVP:  $y'' - 4y' + 13y = 4\delta(t - 10)$ ;  $y(0) = y'(0) = 0$ .
  
3. (14) Express the solution of the following initial value problem in terms of a convolution integral:  
$$y'' - 4y' + 5y = g(t); \quad y(0) = y'(0) = 0.$$
  
4. (14) Solve the IVP:  $X' = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} X$ ,  $X(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
  
5. (a) (10) Find the solution of the following boundary value problem, or show that the solution does not exist:  
$$y'' + 9y = \sin x; \quad y(0) = 0, \quad y'(\pi) = 0$$
  - (b) (4) Find the least period of each of the following functions, or show that the function is not periodic:  $\cos 7x$ ;  $\tan 3x$ ;  $\sinh 2x$ ;  $\sin^2 x$
  
6. Consider the periodic function:  $f(x) = \begin{cases} 2 + x, & -2 \leq x < 0 \\ 2 - x, & 0 \leq x < 2 \end{cases}$   $f(x + 4) = f(x)$ 
  - (a) (4) Sketch the graph of the above function for three periods and state whether the function is odd, even, or neither.
  - (b) (10) Find the Fourier series for the above function.
  
7. Consider the function:  $f(x) = \pi - x$ ,  $0 \leq x < \pi$ .
  - (a) (4) Sketch three periods of the odd  $2\pi$ -periodic extensions of  $f(x)$ .
  - (b) (10) Find the Fourier series for the odd  $2\pi$ -periodic extensions of  $f(x)$ .