Math 222 EXAM III, April 11, 2012

Read each problem carefully. Show all your work for each problem. No Calculators!

1. Seek a power series solution of the equation $y'' + 2xy' + 2y = 0$ about the point $x_0 = 0$:

   (a) (8) Find the recurrence relation.

   (b) (8) Find the first 3 nonzero terms in each of two solutions y_1 and y_2 which form the fundamental set of solutions.

2. (a) (8) Find the general solution of the Euler equation (do not use power series):

$$6x^2y'' + 7xy' - 2y = 0, \quad x>0.$$

   (b) (8) Find all singular points of the equation $x(x+3)y'' - (x-4) - 5y = 0$ and determine whether they are regular or irregular.

3. (a) (8) Find the inverse Laplace transform of $F(s) = \frac{s-2}{s^2+6s+25}$

   (b) (8) Solve the following IVP using the Laplace transform method (no credit will be given for using other methods):

$$y'' + 4y' + 4y = 0, \quad y(0) = -1, \ y'(0) = 2$$

4. (18) Solve the IVP: $\quad y'' + 3y' + 2y = 1+u_2(t) - 2u_3(t)$
   $$y(0) = 0, \ y'(0) = 0$$

5. (18) Solve the IVP: $\quad y'' - 3y' + 2y = g(t), \quad y(0) = 0, \ y'(0) = 0, \quad g(t)=\begin{cases} 1, & 0 \le t < 4 \\ 0, & 4 \le t < \infty \end{cases}$

6. (a) (8) Solve the IVP: $y'' - y = \delta(t-5), \quad y(0) = 1, \ y'(0) = 1$

   (b) (8) Using <u>only</u> the definition of Laplace transform, find the L. transform of $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 1, & 1 \le t < 2 \\ 0, & 2 \le t < \infty \end{cases}$