Math 222 EXAM II, March 7, 2012

Read each problem carefully. Show all your work for each problem! No Calculators!

1.   (16) Find the general solution of each of the differential equations:

    (a) $y'' + 2y' + 17y = 0$ (b) $4y'' + 4y' + y = 0$

2.   (16) Verify that $y_1 = t$ is a solution of the ODE $t^2 y'' + ty' - y = 0$, $t > 0$,
           and find a second linearly independent solution y_2 using the the method of
           reduction of order.

3.   A mass weighing 8 lb stretches a spring 2 ft. Assume there is no damping or external forces
acting on the system. Suppose the mass is pulled down 1 ft below its equilibrium position, and
released with an upward velocity of 4 ft/s.

    (a) (8) Determine the position $y(t)$ of the mass at any time t.

    (b) (8) Find the amplitude, phase angle and period of the motion.

4.   (16) Solve the IVP: $y'' - 2y' + 17y = 0$, $y(\pi/4) = 1$, $y'(\pi/4) = -1$.

5.   Consider the ODE: $y'' - 2y' + 2y = te^t + e^{2t} \sin 2t$

    (a) (6)   Find the fundamental set of solutions of the corresponding homogeneous equation.

    (b) (12) Write a suitable form for the particular solution y_p if the method of undetermined
               coefficients is to be used, but do NOT evaluate the coefficients in the form of y_p.

6.   Consider the ODE: $(1 - t)y'' + ty' - y = 2(t - 1)^2 e^{-t}$, $0 < t < 1$.

    (a) (12)   Assume that $y_1 = e^t$ and $y_2 = t$ form a fundamental set of solutions of the
               corresponding homogeneous equation and find the particular solution y_p by the
               method of variation of parameter.

    (b) (6)   Write the general solution of the above ODE.