Math 222 Exam 3, April 22, 2015

Read each problem carefully. Show all your work for each problem. No calculators!

1. Find power series solutions about $x = 0$ of the equation $2y'' + xy' + 3y = 0$:
   (a) (8 points) Find the recurrence relation.
   (b) (8 points) Find the first four terms in each of two solutions y_1 and y_2 unless the series terminates sooner.

2. (a) (8 points) Find the general solution of the Euler equation and find the solution of the initial value problem
$$x^2y'' - 5xy' + 9y = 0, \quad y(1) = 2, \quad y'(1) = 5.$$
   (b) (8 points) Find all singular points of the given equation and determine whether each is regular or irregular
$$x^2(x-1)^2y'' + 3(x-1)y' + 4y = 0.$$

3. (a) (6 points) Sketch the function, then express it in terms of the unit step function $u_c(t)$, and hence find its Laplace transform
$$f(t) = \begin{cases} 0 & 0 \le t < 1 \\ t-1 & 1 \le t < 3 \\ 0 & 3 \le t. \end{cases}$$
   (b) (6 points) Find the inverse Laplace transform of $F(s) = \frac{s-2}{s^2+2s+5}$.
   (c) (6 points) Find the inverse Laplace transform in terms of a convolution for
$$F(s) = \frac{s}{(s+2)^2(s^2+9)}.$$

4. (16 points) Use Laplace transforms to find the solution of the initial value problem, then sketch the forcing function and the solution versus t
$$y'' + 3y' + 2y = u_2(t), \quad y(0) = 0, \quad y'(0) = 1.$$

5. (16 points) Solve the initial value problem by Laplace transforms, where a is a parameter. Then find the value of a such that $y = 0$ for all $t > \frac{3\pi}{2}$. Sketch the solution for $0 \le t < 2\pi$. (Note that $\sin(t - \frac{3\pi}{2}) = \cos t$)
$$y'' + 4y' + 5y = a\,\delta(t - \frac{3\pi}{2}), \quad y(0) = 1, \quad y'(0) = -2.$$

6. (a) (8 points) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$.
   (b) (10 points) Find the general solution and the solution of the initial value problem for the ODE system $\boldsymbol{x}' = \begin{pmatrix} 3 & -1 \\ 4 & -2 \end{pmatrix} \boldsymbol{x}$, $\boldsymbol{x}(0) = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$. How does the solution behave as $t \to \infty$?