

1. (a) (15 points) Find parametric equations for the line in which the planes  $3x - 6y - 2z = 3$  and  $2x + y - 2z = 2$  intersect.
- (b) (10 points) Find the distance from the point  $(2, -3, 4)$  to the plane  $x + 2y + 2z = 13$ .
2. (a) (10 points) Find the length of the indicated portion of the curve

$$\mathbf{r}(t) = (t \sin t + \cos t)\mathbf{i} + (t \cos t - \sin t)\mathbf{j}, \quad \sqrt{2} \leq t \leq 2.$$

- (b) (15 points) Find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  at  $(1, \ln 2, \ln 3)$  if  $xe^y + ye^z + 2 \ln x - 2 - 3 \ln 2 = 0$ .
3. (a) (10 points) Find the directional derivative of  $f(x, y, z) = x^3 - xy^2 - z$  at  $P_0(1, 1, 0)$  in the direction of  $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ .
- (b) (15 points) Find equations for the tangent plane and normal lines at the point  $P_0$  on the given surface.

$$\cos \pi x - x^2 y + e^{xz} + yz = 4, \quad P_0(0, 1, 2).$$

4. (a) (15 points) Find all the local maxima, local minima, and saddle points of the function

$$f(x, y) = x^4 + y^4 + 4xy$$

- (b) (15 points) Use Lagrange multipliers to find the points on the surface  $xyz = 1$  closest to the origin.
5. (a) (15 points) Sketch the region of integration, change to polar coordinates, and evaluate the integral

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \cos(x^2 + y^2) dx dy.$$

- (b) (15 points) Find the volume of the region bounded by  $z = 4 - 4(x^2 + y^2)$  and  $z = (x^2 + y^2)^2 - 1$ .
6. (20 points) Evaluate the line integral  $\int_C (x + y + z) ds$  over the straight line segment from  $(1, 2, 3)$  to  $(0, -1, 1)$ .

7. (a) (10 points) Show that the vector field  $\mathbf{F} = (2x \ln y - yz)\mathbf{i} + \left(\frac{x^2}{y} - xz\right)\mathbf{j} - xy\mathbf{k}$  satisfies the equations in the Component Test.
- (b) (10 points) Find a potential function for this field. Show all your steps.
- (c) (5 points) Evaluate the line integral  $\int_{(1,2,1)}^{(2,1,1)} \mathbf{F} \cdot d\mathbf{r}$ .
8. (20 points) Apply Green's Theorem to evaluate the counterclockwise circulation of

$$\mathbf{F} = (-y + e^x \ln y)\mathbf{i} + \left(\frac{e^x}{y}\right)\mathbf{j}$$

around the boundary of the region that is bounded above by the curve  $y = 3 - x^2$  and below by the curve  $y = x^4 + 1$ .