

1. (a) (10 points) For the function  $f(x, y) = \sqrt{x - y^2 + 4}$ , find and sketch the domain. Find an equation for the level curve of  $f(x, y)$  passing through the point  $(4, 2)$ .
- (b) (10 points) Find all the second order partial derivatives of  $g(x, y)$ .

$$g(x, y) = y \cos(x) + \ln(x^2 y) + \frac{x^2}{y}.$$

2. (a) (10 points) Find  $\partial w / \partial u$  when  $u = -\pi$ ,  $v = \pi$  if  $w = xy + \ln(z)$ ,  $x = v^2/u$ ,  $y = u + v$ ,  $z = \cos u$ .
- (b) (10 points) Find the directional derivative of  $f(x, y, z) = e^x \sin(4y + 3z)$  at  $P_0(0, 0, 0)$  in the direction of  $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ . Find a unit vector in the direction where the function increases most rapidly at  $P_0$ .
3. (a) (7 points) Find equations for the tangent plane and normal lines at the point  $P_0$  on the given surface.

$$x^2 + xy - y^2 + z = 2, \quad P_0(-1, 1, 3).$$

- (b) (8 points) Find the linearization  $L(x, y, z)$  of the function  $f(x, y, z)$  at the point  $P_0$ .

$$f(x, y, z) = \frac{\sin(xy)}{z}, \quad P_0\left(\frac{\pi}{2}, 1, 1\right).$$

4. (a) (15 points) Find all the local maxima, local minima, and saddle points of the function

$$f(x, y) = 2x^3 + 2y^3 - 3x^2 + 3y^2$$

- (b) (10 points) Find the maximum and minimum values of  $f(x, y, z) = 5x + y - 2z$  on the sphere  $x^2 + y^2 + z^2 = 30$ .
5. (a) (8 points) evaluate the double integral over the given region  $R$ .

$$\iint_R \frac{3xy^2}{x^2 + 1} dx dy, \quad R: 0 \leq x \leq 1, 0 \leq y \leq 2.$$

- (b) (12 points) Sketch the region of integration, reverse the order of integration, and evaluate the integral

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} dy dx.$$