Math 211 Exam 1 Spring 2015

1. Given the points $P(2,0,0)$, $Q(0,4,1)$ and $R(3,3,0)$ in space.
   (a) (10 points) Find the vector $\overrightarrow{V} = 2(\overrightarrow{PQ} - \overrightarrow{QR})$.
   (b) (10 points) Find the vector of length 6 in the direction opposite to $\overrightarrow{PR}$.
2. Given the points $P(1,2,3)$, $Q(0,2,5)$ and $R(4,3,4)$ in space,
   (a) (10 points) Find the area of the triangle $\triangle PQR$ using a cross product.
   (b) (10 points) Find the cosine of vertex angles at P, Q and R.
3. Given the points $P(1,0,1)$, $Q(2,0,0)$ and $R(-1,1,0)$ in space,
   (a) (5 points) ind an equation of a plane through the points P, Q, and R.
   (b) (5 points) Where does the line $x = 1 + 2t$, $y = 2 + t$, $z = -1 - t$ intersect the plane that contains P, Q, and R?
   (c) (10 points) Find a parametric representation of the line of intersection of the plane $3x - 6y + 2z = 4$ and the plane that contains P, Q, and R.
4. Sketch each of the following surfaces and find their intersection points (if any) with the coordinate (x, y and z) axes,
   (a) (5 points) $z = y^2 - 1$.
   (b) (10 points) $z = 8 - x^2 - y^2$.
5. The position vector of a particle moving through space is

$$\mathbf{r}(t) = (t - \sin t)\,\mathbf{i} + (1 - \cos t)\,\mathbf{j} + t\,\mathbf{k},\ t \geq 0.$$

   (a) (5 points) Find the velocity and acceleration vectors and the speed as a function of t.
   (b) (5 points) Find the parametric equation of the tangent line to the curve described by the particle at $t = \pi/4$.
   (c) (10 points) Find the time(s) (if any) when the particle's acceleration vector is orthogonal to its velocity vector.