Math 111 – Spring 2014
Final Examination

Please complete the following problems. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not allowed during this examination.

1.(18 pts.) Evaluate the following limits:

a. $\lim_{x\to 2^-} (x-2)\tan\left(\frac{\pi x}{4}\right)$ **b.** $\lim_{x\to 0^+} x^{\frac{1}{x}}$

c. $\lim_{x\to\infty} x^{\frac{1}{\sqrt{x}}}$

2.(18 pts.) Find $y'(x)$ for the following:

a. $y = \tan(x)\,\arctan(x)$ **b.** $y = \sin(xy) + e^y$

c. $y = (\cos(x))^x$

3.(18 pts.) Find $y'(x)$ for the following:

a. $y = \dfrac{x^3}{x + \ln(x)}$ **b.** $y = \displaystyle\int_{\sqrt{3}}^{x^3} \cos(t^2)\,dt$

c. $y = \cos^2(\sec(2^x))$

4.(21 pts.) Evaluate the following integrals:

a. $\displaystyle\int_1^2 \left(\cos\left(\frac{\pi x}{2}\right) + \frac{2}{x}\right) dx$ **b.** $\displaystyle\int \frac{1+x}{\sqrt[3]{x}}\,dx$

c. $\displaystyle\int_{-1}^1 \frac{x}{1+x^2}\,dx$

5.(14 pts.) Evaluate the following integrals:

a. $\displaystyle\int_0^\pi \frac{\sin(x)}{2+\cos(x)}\,dx$ **b.** $\displaystyle\int x\sec^2(x^2)\,dx$

6.(14 pts.) Evaluate the following integrals:

a. $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$ **b.** $\displaystyle\int x\sqrt{x-2}\,dx$

7.(10 pts.) Find the total area between $y = e^x$ and $y = e^{-x}$ for $-1 \le x \le 2$.

8.(10 pts.) Find the area of the region enclosed by $y = x^2 + x - 1$ and $y = 2x + 1$.

9.(20 pts.) This question is about tangents and normals to curves.

a. Find the tangent line to $y = (\arcsin(x))^2$ at $x = \frac{1}{2}$.

b. Find the normal line to $y = \sqrt{1 + \sqrt{x}}$ at $x = 1$.

10.(11 pts.) Find the right triangle with hypotenuse 2 ft which has the greatest area using calculus methods. Show that your result is a maximum.

11.(11 pts.) A spherical balloon is inflated so that its volume is increasing at the rate 6 cm^3/min. At what rate is the radius increasing when the radius is 3 cm?

12.(35 pts.) Consider the function $y = \dfrac{x^2 + 4}{x^2 - 4}$.

a. Find all asymptotes of this function.

b. Find the intervals on which this function is increasing or decreasing.

c. Find the intervals on which this function is concave up or concave down.

d. Determine the points (if any) at which this function has a local maximum, a local minimum, or a point of inflection.

e. Sketch a graph of this function making sure to label the asymptotes from part **a** and the points found in part **d**.