Math 111 – Fall 2014
Final Examination

Please complete the following problems. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not allowed during this examination.

1.(21 pts.) Evaluate the following limits:

a. $\lim_{x\to 2} \dfrac{x^2-4}{\sqrt{x^2+5}-3}$

b. $\lim_{x\to 0^+} x^{-\frac{1}{\ln(x)}}$

c. $\lim_{x\to 0^+} \left(\dfrac{1}{\sin(x)} - \dfrac{1}{\sin(2x)}\right)$

2.(21 pts.) Evaluate the following limits:

a. $\lim_{x\to 1^-} \dfrac{\sqrt{1-x^2}}{\arcsin(x)}$

b. $\lim_{x\to \pi^-} \sin(x)^{\csc(x)}$

c. $\lim_{x\to 1} \dfrac{\sin(\pi x)}{\ln(x)}$

3.(21 pts.) Find $y'(x)$ for the following:

a. $y = x^3 \arcsin(5x)$

b. $y = x^{2x} + x^2$

c. $y = \ln(y\cos(x))$

4.(21 pts.) Find $y'(x)$ for the following:

a. $y = \dfrac{\sin(\sqrt{x})}{\sqrt{x}}$

b. $y = \displaystyle\int_{x^3}^{10} \sin(t^2)\,dt$

c. $y = \tan\big(\sec^2\big(3x^2\big)\big)$

5.(21 pts.) Evaluate the following integrals:

a. $\displaystyle\int \left(\frac{1}{x} + \frac{1}{x\ln(x)}\right) dx$

b. $\displaystyle\int_1^4 \frac{1+x^{\frac{3}{2}}}{x^{\frac{1}{2}}}\,dx$

c. $\displaystyle\int_{-\pi}^{\pi} \frac{\sin(x)}{4+\cos(x)}\,dx$

6.(14 pts.) Evaluate the following integrals:

a. $\displaystyle\int_0^1 \frac{e^x}{1+e^x}\,dx$

b. $\displaystyle\int x\sin\left(\frac{x^2}{4}\right) dx$

7.(14 pts.) Evaluate the following integrals:

a. $\displaystyle\int \frac{x+2}{x^2+1}\,dx$

b. $\displaystyle\int x^3\sqrt{x^2+4}\,dx$

8.(13 pts.) Find the area of the region enclosed by $y = 2x^2+2$ and $y = 3x+1$.

9.(13 pts.) A plane flying horizontally at an altitude of 1 mile and a constant speed of 500 miles/hr passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when the plane is 2 miles away from the station.

10.(13 pts.) Consider the two curves $y = x^2$ and $y = x^3$ for $0 \leq x \leq 1$. Find the length of the longest vertical line segment between these curves. Show that your result is a maximum.

11.(28 pts.) Consider the function $y = x\sqrt{3+x}$.

a. Find the intervals on which this function is increasing or decreasing.

b. Find the intervals on which this function is concave up or concave down.

c. Determine the points at which this function has a local maximum, a local minimum, or a point of inflection.

d. Sketch a graph of this function making sure to label the points found in part **c**.