Math 111 – Fall 2014
Examination 1

Please complete the following problems. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not allowed during this examination.

1.(10 pts.) Evaluate the following limits, allowing $+\infty$ and $-\infty$ as possible values of a limit. If the limit does not exist, explain why.

   a. $\displaystyle\lim_{x\to 1}\left(\frac{1}{x+1}-\frac{1}{x^2+1}\right)$ **b.** $\displaystyle\lim_{x\to 0^-}\arctan\left(\frac{1}{x}\right)$

2.(10 pts.) Evaluate the following limits, allowing $+\infty$ and $-\infty$ as possible values of a limit. If the limit does not exist, explain why.

   a. $\displaystyle\lim_{x\to 2}\frac{x^2-4}{2-\sqrt{x+2}}$ **b.** $\displaystyle\lim_{x\to 0}\frac{\sin(3x)}{\tan(2x)}$

3.(10 pts.) Evaluate the following limits, allowing $+\infty$ and $-\infty$ as possible values of a limit. If the limit does not exist, explain why.

   a. $\displaystyle\lim_{x\to 0}\frac{\sqrt{x+1}}{x-\sqrt{x^2+1}}$ **b.** $\displaystyle\lim_{x\to\infty}\arcsin\left(e^{\frac{1}{x}}\right)$

4.(10 pts.) Evaluate the following limits, allowing $+\infty$ and $-\infty$ as possible values of a limit. If the limit does not exist, explain why.

   a. $\displaystyle\lim_{x\to 0^+}\frac{1}{1-\cos(x)}$ **b.** $\displaystyle\lim_{x\to 1^+}\frac{|1-x|}{x^2-4x+3}$

5.(12 pts.) Use the definition of the derivative as the limit of a difference quotient to find the derivative of $f(x)=\dfrac{1}{x^2}$.

6.(20 pts.) Find all horizontal, vertical, and slant (oblique) asymptotes, if they exist, for the following functions. Be sure to clearly label the type of each of your asymptotes.

   a. $y=\dfrac{x^2-x}{x-2}$ **b.** $y=\dfrac{2\sqrt[3]{x}+1}{\sqrt[3]{x}+2}$

7.(16 pts.) Find all points where each of the following functions is discontinuous and identify the type of discontinuity.

a. $y = \dfrac{1}{|x| - 1} + \dfrac{1}{|x| + 2}$
   b. $y = \dfrac{1}{|x - 1|} + \dfrac{1}{|x + 2|}$

8.(12 pts.) Find all points where f is discontinuous and identify the type of discontinuity.

$$f(x) = \begin{cases} 2 - x, & x \leq 1 \\ \dfrac{1}{\ln(x) - 1}, & x > 1 \end{cases}$$