Fall 2018   December 19, 2018
MATH-333 (Common Final Exam)   NJIT

Q. # 1	#2	#3	#4	#5	#6	#7	Total
13	13	9	13	15	16	21	100

This exam is closed book and closed notes. Basic calculator is allowed.
Formula sheet and tables are provided with the exam. You will need to show work to get full credit.
Use of advanced/programmable/graphing calculators and other electronic devices is not permitted during the exam.

Name (PRINT) _____   Section # _____
                    Last   First

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Problem 1) This problem has three independent parts a), b), and c)

a) (5 points) It is known that the scores on a certain test are normally distributed with standard deviation σ. For a random sample of 20 students, the average score on the test was found to be 63.4 with a standard deviation of 12.34. Construct a 90% upper confidence bound for σ. **(Round your answer to two decimal places)**

b) (3 points) (Circle only one) Which of the following measure is least affected by outliers?

   i) Mean   ii) Standard deviation   iii) Interquartile range   iv) Range

c) **(5 points)** Suppose that $P(A|B)=0.2$, $P(A|B')=0.3$, $P(B)=0.8$. Find $P(A)$. **(Round your answer to two decimal places)**

Problem 2) This problem has two independent parts a) and b)

a) The cumulative distribution function of the random variable X, denoting the number of episodes of otitis media in the first two years of life is given below:

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.129 & 0 \le x < 1 \\ 0.393 & 1 \le x < 2 \\ 0.664 & 2 \le x < 3 \\ 0.849 & 3 \le x < 4 \\ 0.944 & 4 \le x < 5 \\ 0.983 & 5 \le x < 6 \\ 1 & 6 \le x \end{cases}$$

i) (5 points) Determine the probability mass function (pmf) of X, f(x).

ii) (4 points) Determine $P(3 \le X \le 6)$. **(Round your answer to three decimal places).**

**b) (4 points)** Let the random variable X denote the shaft clearance in millimeters in a sheet metal component. The probability density function of X is given by

$f(x) = 1.25(1-x^4)$ $0 < x < 1$, $f(x) = 0$ elsewhere.

Obtain the cumulative distribution function (cdf) of X, F(x)

Problem 3) This problem has two independent parts a) and b)

a) **(4 points)** The boxplots below summarize the distribution of SAT verbal and math scores among students at a high school.
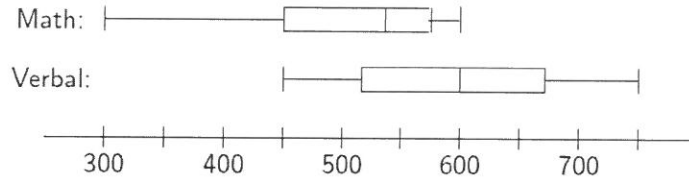
Which of the following statements are true?
I. The range of the math scores equals the range of the verbal scores.
II. The highest math score equals the median verbal score.
III. The verbal scores appear to be roughly symmetric, while the math scores appear to be skewed to the right.
(Circle only one of the following)
(A) I only
(B) III only
(C) I and II
(D) II and III
(E) I, II, and III

b) **(5 points)** Suppose scores of students on a test are approximately normally distributed with a mean score of 65 points and a standard deviation of 8 points. It is decided to give A's to 10 percent of the students. Obtain the threshold score that will result in an A. Show work to get full credit. **(Round your answer to two decimal places)**

Problem 4) This problem has three independent parts a), b), and c)

a) **(5 points)** Customers arrive at a service window according to a Poisson process with an average of 0.2 per minute. What is the probability that the time between two successive arrivals is less than 6 minutes? **(Round your answer to four decimal places)**

b) **(5 points)** The volume of a shampoo filled into a container is uniformly distributed between 374 and 380 milliliters. What is the volume of shampoo that is exceeded by 95% of the containers?

c) **(3 points)** Quiz scores of 50 students from a statistics class are tabulated and the standard deviation of the scores is calculated (S_{old}). If you now add 5 to all the tabulated scores and calculate the new standard deviation (S_{new}), how does it change the new standard deviation? Choose the one that applies:

   i) The new standard deviation increases by 5, i.e., $S_{new} = S_{old} + 5$

   ii) The new standard deviation is 15 times the old standard deviation, i.e., $S_{new} = 5 * S_{old}$

   iii) The new standard deviation remains unchanged, i.e., $S_{new} = S_{old}$

   iv) The new standard deviation becomes zero, i.e., $S_{new} = 0$

PLEASE SHOW WORK TO GET FULL CREDIT

Problem 5) This problem has three independent parts a), b), and c)
The height of plants is normally distributed with a mean of 32 inches and a standard deviation of 8 inches.
  a) (5 points) What is the probability that the average height of a sample of 25 plants is greater than 30 inches? **(Round your answer to three decimal places)**

b) (6 points) If a random sample of 5 plants is selected, what is the probability that none of these 5 plants has height exceeding 30 inches? **(Round your answer to four decimal places)**

c) (4 points) In a test of hypothesis at $\alpha=0.01$, the null hypothesis is that the population mean is equal to 60 and the alternative hypothesis is that the population mean is not equal to 60. A sample of 36 elements selected from this population produced a mean of 63. Given $\sigma = 6.3$. What is the approximate p-value for this test? **(Circle only one)**

  i) 0.0347

  ii) 0.0042

  iii) 0.0021

  iv) 0.0952

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Problem 6) This problem has three independent parts a), b, and c)

a) (5 points). The life in hours of a 75-watt light bulb is known to be normally distributed with $\sigma=25$ hours. A random sample of 25 bulbs has a mean life of 1014 hours. Construct a 99% lower confidence bound on the mean life. **(Round your answer to two decimal places)**

b) (5 points) Suppose a company wishes to determine the proportion of parts produced that have visible defects. It is estimated that 7% of all parts have defects. How large of a sample must be taken if it is desired to estimate the proportion to within 3% using a 90% confidence interval?

c) (6 points) A manufacturer of athletic shoes claims that the mean life of his shoes will exceed 50 hours. The shoe life is known to be normally distributed with a standard deviation $\sigma = 5$ hours. A researcher wishes to test the manufacturer's claim with a sample of 25 shoes at $\alpha=0.01$. Compute the power of the test if the true mean life $\mu = 54$ hours. (Note that you only have to compute the power of the test in this problem) **(Round your answer to four decimal places)**

Problem 7) This problem has three independent parts a), b), and c)

a) **(6 points)** The burning rates of two different solid-fuel propellants used in air crew escape systems are being studied. It is known that both propellants have approximately the same standard deviation of burning rate: $\sigma_1 = \sigma_2 = 3$ centimeters per second. Two random samples of $n_1 = 36$ and $n_2 = 36$ specimens are tested; the sample mean burning rates are $\overline{x}_1 = 24$ centimeters per second and $\overline{x}_2 = 18$ centimeters per second. Construct a 98% confidence interval on the difference in means. Would you conclude that the means are different based on the confidence interval obtained? **(Round your answer to two decimal places)**

Conclusion (circle only one)
i) Based on the confidence interval obtained we conclude that the means are different
ii) Based on the confidence interval obtained we conclude that the means are not different.

b) **(5 points)** We have examined the age (x) and price (y) of a certain model of car. The age (in years) and price (in dollars) for a sample of 15 such cars yielded the following equation: $\hat{y} = 18600 - 1800x$
Predict the price of a five year old car of that model. **(Circle only one)**

i) $9400
ii) $9600
iii) $9800
iv) $16800

c) A manufacturer claims that the mean lifetime of its fluorescent bulbs is more than 900 hours. A homeowner selects a random sample of 16 bulbs and finds the mean lifetime to be 920 hours with a standard deviation of 70 hours. Assume that the lifetime of fluorescent bulbs is normally distributed. State the null and alternative hypotheses. Is there enough evidence to support the manufacturer's claim? Test using rejection regions (critical regions) at α=0.05.

Null and Alternative Hypotheses (2 points):

Critical value(s) (2 points):

Test Statistic (2 points):

Interpretation and Conclusion:
Circle only one (1 point):
i) Reject the null hypothesis
ii) Fail to reject the null hypothesis

Why? (2 points)

Is there enough evidence to support the manufacturer's claim? **Circle only one (1 point):**
i) Yes, there is enough evidence at α=0.05 to support the manufacturer's claim
ii) No, there is not enough evidence at α=0.05 to support the manufacturer's claim

Extra Space (ANY "Rough Work" must be crossed out)