Math 111 Exam #1
Sept. 27, 2017

Time: 1 hour and 10 minutes
Instructions: Show all work for full credit.
No outside materials or calculators allowed.
Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: ____________________

ID #: ____________________

Instructor/Section: ____________________

Problem	Value	Score
1	15 pts.	
2	20 pts.	
3	20 pts.	
4	20 pts.	
5	15 pts.	
6	10 pts.	
TOTAL	100	

"I pledge by my honor that I have abided by the NJIT Academic Integrity Code."

____________________ (Signature)

1. Consider the curve $y = f(x) = x^3 + x + 1$.

(a) Explain (using a theorem) why $f(x) = 0$ for some x in the interval $[-1,1]$. **(5 pts.)**

(b) Find the tangent line to the curve at $(0,1)$. **(10 pts.)**

(a) $\left.\begin{array}{l} f(-1) = -1-1+1 = -1 < 0 \\ f(1) = 1+1+1 = 3 > 0 \end{array}\right\} \Rightarrow 0 \in [f(-1), f(1)]$

f is continuous as it is a polynomial.

The Intermediate Value Theorem requires $f(c) = 0$ for some $c \in [-1,1]$.

(b) $f'(x) = 3x^2+1 \Rightarrow f'(0) = 1$

The tangent line is given by

$y - 1 = 1 \cdot (x - 0)$

or $\boxed{y = x + 1}$

2. Evaluate the following limits, allowing $+\infty$ and $-\infty$ as possible values of a limit. If the limit does not exist, explain why. Show all work. **(5 pts. each)**

(a) $\lim_{x\to 0}\dfrac{\tan(3x)}{\sin(x)}$ (b) $\lim_{x\to 4}\dfrac{4-x}{5-\sqrt{x^2+9}}$ (c) $\lim_{x\to\infty}\left(e^{-x}+\tan^{-1}(x)\right)$ (d) $\lim_{x\to 2}\dfrac{x^3-4x}{x^2-2x}$.

(a) $\lim_{x\to 0}\dfrac{\tan(3x)}{\sin(x)} = \lim_{x\to 0}\dfrac{3x}{x}\cdot\dfrac{x}{\sin(x)}\cdot\dfrac{\sin(3x)}{3x}\cdot\dfrac{1}{\cos(3x)}$

$= 3\cdot 1\cdot 1\cdot 1 = \boxed{3}$

(b) $\lim_{x\to 4}\dfrac{4-x}{5-\sqrt{x^2+9}}\cdot\dfrac{5+\sqrt{x^2+9}}{5+\sqrt{x^2+9}} = \lim_{x\to 4}\dfrac{(4-x)\left(5+\sqrt{x^2+9}\right)}{25-(x^2+9)}$

$= \lim_{x\to 4}\dfrac{(4-x)\left(5+\sqrt{x^2+9}\right)}{16-x^2} = \lim_{x\to 4}\dfrac{(4-x)\left(5+\sqrt{x^2+9}\right)}{(4-x)(4+x)}$

$= \lim_{x\to 4}\dfrac{5+\sqrt{x^2+9}}{4+x} = \dfrac{5+5}{4+4} = \boxed{\dfrac{5}{4}}$

(c) $\lim_{x\to 2}\dfrac{x^3-4x}{x^2-2x} = \lim_{x\to 2}\dfrac{x(x-2)(x+2)}{x(x-2)} = \lim_{x\to 2} x+2 = \boxed{4}$

3. For what values of a and b is the function defined as

$$f(x) = \begin{cases} (x^2 + 2x - 15)/(x-3), & x < 3 \\ ax + b, & 3 \le x \le 5. \\ 0, & 5 < x \end{cases}$$

continuous on the whole real line $(-\infty, \infty)$? Show all work. **(20 pts.)**

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2 + 2x - 15}{x-3} = \lim_{x \to 3^-} \frac{(x-3)(x+5)}{x-3} = 3 + 5 = 8$$

$f(3) = 3a + b$

$f(5) = 5a + b$

$\lim_{x \to 5^+} f(\ ) = 0$

$\Rightarrow 3a + b = 8$ and

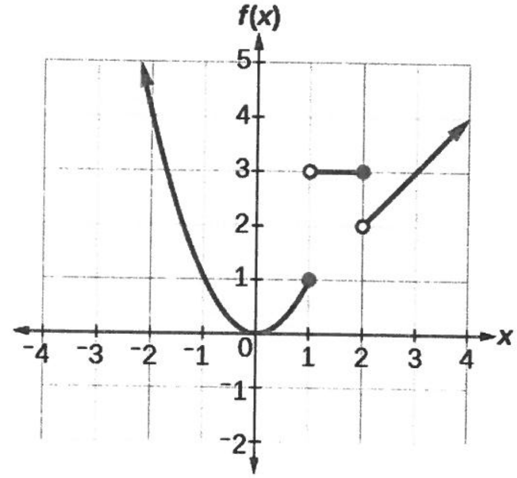
$5a + b = 0$

So $2a = -8 \Rightarrow \boxed{a = -4}$ and $5 \cdot (-4) + b = 0$ or $\boxed{b = 20}$

4. Given the graph of the piecewise function $f(x)$, answer the following: **(10 pts. each)**

a. Find $\lim_{x \to 1} f(x)$ or explain why it does not exist (Show all work, including left and right limits)

b. Find f'(2.5), the derivative of the function at x = 2.5



ⓐ $\lim_{x \to 1} f(x)$ does not exist

because $\lim_{x \to 1^-} f(x) = 1$ but $\lim_{x \to 1^+} f(x) = 3$

ⓑ $f'(2.5) = 1$ since $f(x) = x$ for $x > 2$

5. Find all horizontal, vertical and slant (oblique) asymptotes for the following function. Show all work involving limits and other methods. **(15 pts.)**

$$f(x) = \frac{x^2 - x}{x - 2}.$$

Horizontal asymptotes

$\boxed{\text{none}}$ $\quad \lim_{x \to \infty} f(x) = +\infty$ and $\lim_{x \to -\infty} f(x) = -\infty$

Vertical asymptotes.

$\boxed{x = 2}$ $\quad (x - 2 = 0 \Rightarrow x = 2)$

Slant asymptotes

$$\begin{array}{r|l} & x + 1 \\ \hline x - 2 & x^2 - x \\ & x^2 - 2x \\ \hline & x \\ & x - 2 \\ \hline & 2 \end{array}$$

$$\Rightarrow f(x) = x + 1 + \frac{2}{x - 2}$$

$$\boxed{y = x + 1}$$

6. Use the limit quotient definition to find the derivatives of each of the following functions, and show all work:

(a) $y = f(x) = x^2 + 1$ **(4 pts.)**

(b) $y = g(x) = (x+1)^{-1}$, for $x \neq -1$. **(6 pts.)**

(a)
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{h}$$
$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+1} - \cancel{x^2} - \cancel{1}}{h}$$
$$= \lim_{h \to 0} 2x + h = \boxed{2x}$$

(b)
$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$
$$= \lim_{h \to 0} \frac{(x+h+1)^{-1} - (x+1)^{-1}}{h}$$
$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{1}{x+h+1} - \frac{1}{x+1} \right]$$
$$= \lim_{h \to 0} \frac{1}{h} \left[\frac{(x+1) - (x+h+1)}{(x+h+1)(x+1)} \right]$$
$$= \lim_{h \to 0} \frac{1}{\cancel{h}} \frac{-\cancel{h}}{(x+h+1)(x+1)} = \boxed{\frac{-1}{(x+1)^2}}$$