Math 112 Final Exam

December 19, 2016

	Problem(s)	Score	Total
Time: 2 hour and 30 minutes			
Instructions: Show all work for full credit.			
No outside materials or calculators allowed.			
Extra Space: Use the backs of each sheet			
for extra space. Clearly label when doing so.			
Name:			
ID #:			
Instructor/Section:			
"I pledge by my honor that I have abided by the NJIT Academic Integrity Code."			
(Signature)			

- 1. Consider the function $f(x) = \frac{1}{x}$.
 - a. Find the full Taylor series representation of f(x) centered around x=2 (6 points)



2. Determine if the following series converge or diverge. Show all work including stating any tests for convergence. (10 points)

$$a) \sum_{n=0}^{\infty} \frac{n10^n}{(2n)!}$$

$$b) \sum_{n=1}^{\infty} \frac{\ln(n) + n}{n^2 + 4}$$

3. Suppose the curve $y=x^2$, $0 \le x \le 2$ *meters* is revolved around the y-axis to create a tank. Find the work done in filling this tank with a liquid with density 3000 N/m³ pumped up from the x-axis. (7 points)



4. Find the volume of the figure formed by rotating the region bound by y=sinx, x=0, x= π

and y=0 about the y-axis. (7 points)

5. Integrate the following (10 points):

a.
$$\int \frac{x^2}{(1-x^2)^{3/2}} dx$$

b.
$$\int \sin^3(x)\cos^2(x)dx$$

6. Integrate the following (10 points):

a.
$$\int x^2 \sinh(2x^3) dx$$

b.
$$\int \frac{x+4}{x(x+2)^2} dx$$

7. Integrate the following (10 points): a. $\int x \tan^{-1}(x) dx$

a.
$$\int x \tan^{-1}(x) dx$$

 $b. \int_0^1 \frac{1}{\sqrt{x^3}} dx$

8. For each of the following, circle <u>ALL VALUES</u> of k for which the following converge (note, there may be multiple answers circled). For this problem, only your answers will be graded, not the quality of your scratch work if needed (6 points).

a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(k+2)n^k}$$

- a) k = 3
- b) k = 1
- c) $k = \frac{1}{2}$
- d) k = 0
- e) k = -1
- f) none of these

$$b) \sum_{n=1}^{\infty} \frac{k}{n^k}$$

- a) k = 3
- b) k = 1
- c) $k = \frac{1}{2}$
- $d) \quad k = 0$
- e) k = -1
- f) none of these

c) the sequence
$$\left\{\frac{1}{n^k}\right\}_{n=1}^{n=\infty}$$

- g) k = 3
- h) k = 1
- i) $k = \frac{1}{2}$
- j) k = 0
- k) k = -1
- 1) none of these

- 9. Consider the parametric equation given by x(t) = cos(2t), y(t) = sin(2t).
 - a. Find the length of this curve on the range $0 \le t \le \frac{\pi}{2}$ (4 points)

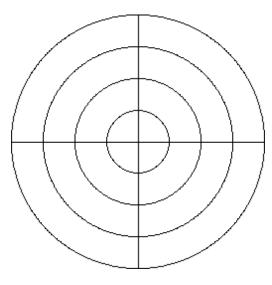
b. Find the equation of the tangent line to the curve when $t = \pi/8$ (4 points)

c. Graph the portion of the parametric curve that you found arc length for in part (a) and the tangent line found in part (b) (3 points)

- 10. Consider the function $f(x) = \frac{3}{4}x^{4/3}$ (8 points) a. Find P₂(x), the 2nd degree Taylor Polynomial centered around x=8 for f(x)

b. Suppose f(x) is approximated by the Taylor Polynomial from part (a). Use Taylor's Theorem to bound the error when $1 \le x \le 8$.

11. Graph the polar curves $r = 1 + \cos(\theta)$ and $r = 3\cos(\theta)$ on the axis below (3 points):



b) Find the area inside $r = 3\cos(\theta)$ but outside $r = 1 + \cos(\theta)$ (7 points):