Math 110 Final Exam
May 5, 2017

Time: 2 hour and 30 minutes
Instructions: Show all work for full credit.
No outside materials or calculators allowed.
Extra Space: Use the backs of each sheet
for extra space. Clearly label when doing so.

Name: _____
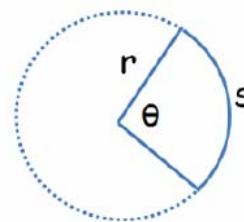
ID #: _____

Instructor/Section: _____

*"I pledge by my honor that I have abided by the
NJIT Academic Integrity Code."*

_____ (Signature)

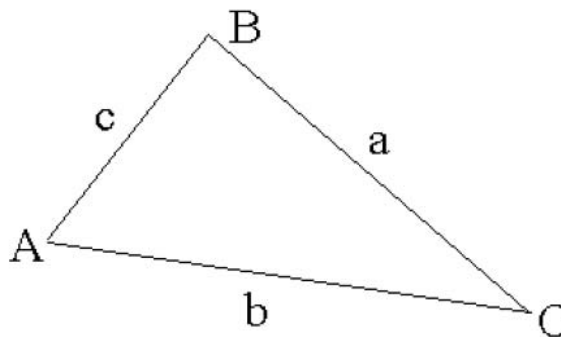| Problem(s)	Score	Total

Circular motion and equations relating to a sector of a circle,
radius r (as shown to the right).

$A = \frac{1}{2}\theta r^2$ (where A is the area of the sector cut out by θ)
$s = r\theta$ (where s is the arc length as shown)
$v = \omega r$ (where v is velocity and ω is angular velocity)

$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$
$\sin(A-B) = \sin(A)\cos(B) - \sin(B)\cos(A)$
$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

Given ΔABC as shown to the right:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

$a^2 = b^2 + c^2 - 2bc\cos(A)$
$b^2 = a^2 + c^2 - 2ac\cos(B)$
$c^2 = b^2 + a^2 - 2ab\cos(C)$
$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$
$\text{Area} = \frac{1}{2}ab\sin C$

Show all your work. Simplify and reduce all answers as much as possible. Rationalize all denominators.

1. **(12 pts)** Find the exact value of the following, or state that it is undefined.

   a) $\cos\left(-\frac{7\pi}{4}\right)$

   b) $\sin\left[\sin^{-1}\left(\frac{1}{5}\right)\right] + \log_2\sqrt{8}$

   c) $\sin(1020°)$

   d) $\tan[\cos^{-1}(x)]$, give your answer in terms of x

2. **(4 pts)** Suppose that a pulley with a radius of 10 inches is rotating such that a point on the rim of the pulley is moving at a velocity of 2 feet/minute. Find how many revolutions per minute this point is making.

3. Sketch the graph of the equation on the given set of axes below by making a table of values or using transformations. Identify and label any asymptote(s).

a) **(4 pts)** $y = -\log_2(x + 2)$
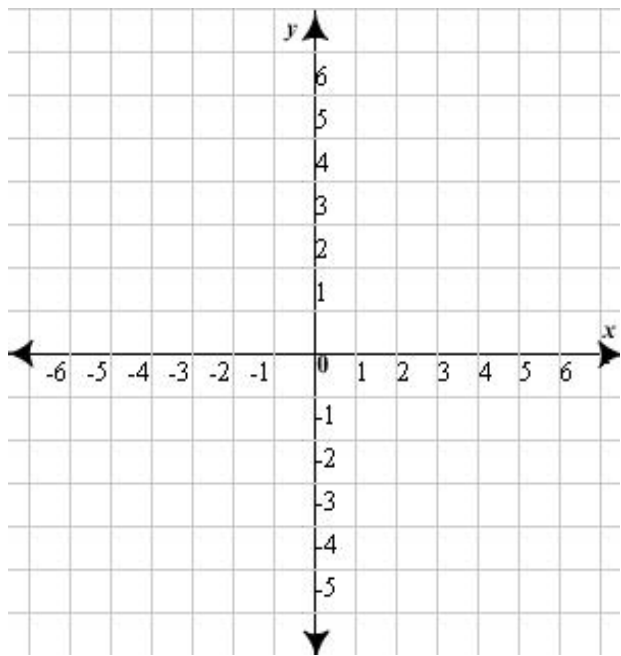

b) **(4 pts)** $y = 2^{-x} - 1$
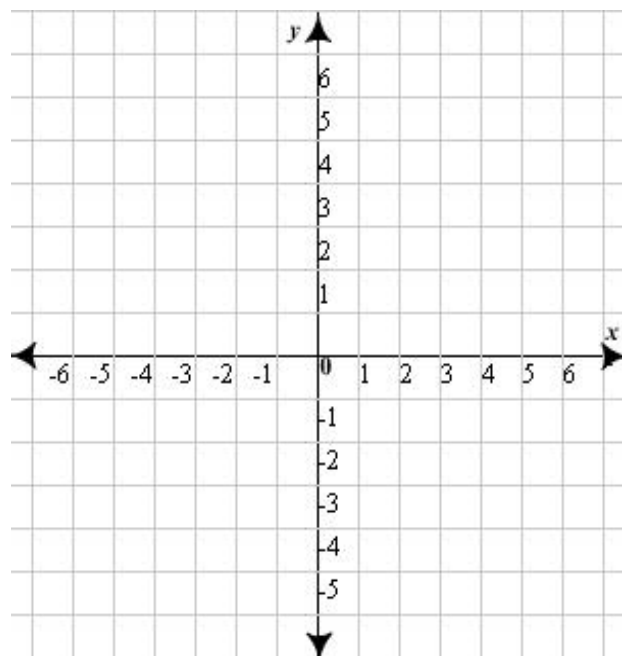
4. **(5 pts)** Write a partial fraction decomposition for the rational expression: $\dfrac{3x^2+4x+4}{x^3-4x}$.

5. a) **(5 pts)** Find the point(s) where the curve $y^2 = -2x$ intersects the circle $x^2 + y^2 = 3$.

b) **(3 pts)** Convert the polar equation: $r = 3\cos^2(\theta) + 3\sin^2(\theta)$ into a rectangular equation.

6. a) **(6 pts)** First convert the equation $4(x + 1)^2 + (y - 2)^2 = 16$ into its standard form. Then graph the equation on the set of axes below. Be sure to find and label the center and the vertices on the graph.
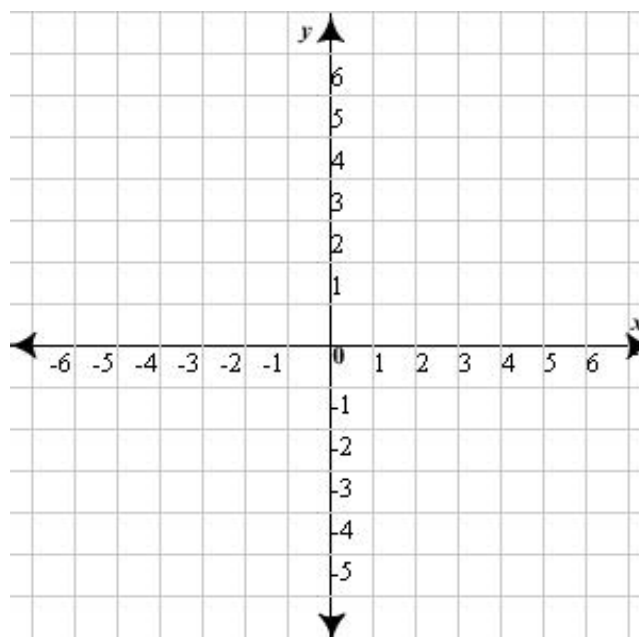
b) **(4 pts)** Graph one period of the equation below on the set of axes by finding the amplitude, period and the quarter points. Be sure to label the 5 key points on the graph.

$$y = -4 \sin\left(\frac{\pi}{2}x\right)$$
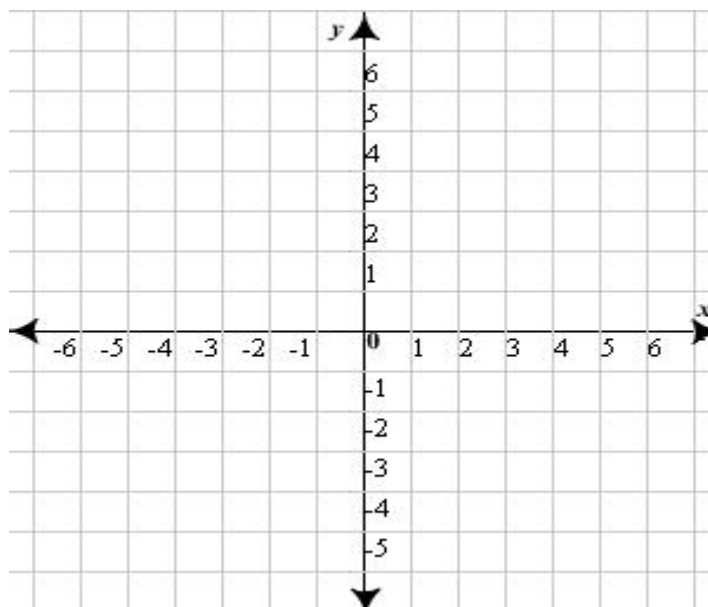
7. **(9 pts)** Suppose $\cos\alpha = \dfrac{4}{5}$, with α in Quad. I, and $\sin\beta = -\dfrac{5}{13}$, with β in Quad. IV, find:

a) $\sin(2\beta)$

b) $\cos\left(\dfrac{\alpha}{2}\right)$

c) $\sin(\alpha + \beta)$

8. a) **(5 pts)** If $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & 5 \end{bmatrix}$, solve the matrix equation for the unknown matrix X:

$$4B + X = BA - 3A$$

b) **(6 pts)** A system of linear equations is given below in the form of a matrix equation alongside the associated augmented matrix. Solve the system of equations using any method discussed in class.

$$\begin{bmatrix} 1 & -3 & 2 \\ 2 & 4 & -3 \\ 3 & -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -9 \\ 12 \end{bmatrix}$$ which has the augmented matrix $\left[\begin{array}{ccc|c} 1 & -3 & 2 & 9 \\ 2 & 4 & -3 & -9 \\ 3 & -2 & 5 & 12 \end{array}\right]$
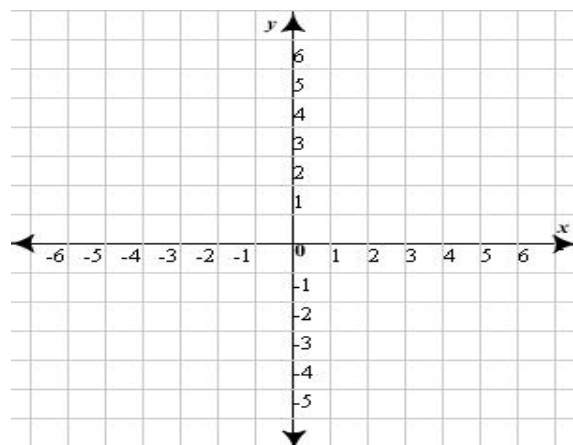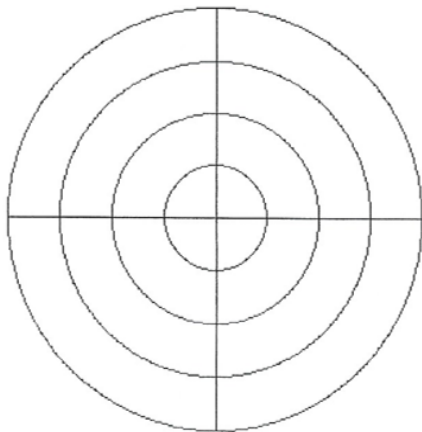
9. **(6 pts)** Solve the following equation for ***all solutions*** in the domain.

a) $\log(x - 1) + \log(x + 2) = 1$

b) $2\sin(x)\cos(x) = \sin(x)$, ***for all x in*** $[0, 2\pi)$

10. **(5 pts)** Sketch the graph of the polar curve: $r = 3\cos(2\theta)$ on either of the axes given below.

11. **(3 pts each)** The total net profit for a certain company throughout the year can be modeled by the function: $P(t) = 10,000 + 20,000 \cos\left[\frac{\pi}{6}t - \frac{11\pi}{6}\right]$, where $P(t)$ is the total profit in dollars generated during month, with $t = 0$ corresponding to January through $t = 11$ corresponding to December.

a) Find the total net profit for the month of February.

b) Determine the maximum net profit that this company could generate in any month.

c) During which month(s) is this maximum net profit attained?

d) During which month(s) does the net profit exactly equals $0?

12. **(5 pts)** Suppose a triangle $\triangle ABC$ has a side lengths of $a = 5$ and $b = 8$ while $\angle C = 60°$. Use this information to solve for the length of side c and the area of the triangle.

13. a) **(3 pts)** Given the function $f(x) = x^2 - x$, evaluate and simplify the quantity: $\frac{f(x+h)-f(x)}{h}$.

b) **(2 pts)** Given $f(x) = x^{\frac{2}{3}}(\sqrt{x})$, find $f(27)$.