Math 110 Common Exam II
March 8, 2017

Time: 1 hour and 25 minutes

Instructions: Show all work for full credit.
No outside materials or calculators allowed.

Extra Space: Use the backs of each sheet
for extra space. Clearly label when doing so.

Name: _____

ID #: _____

Instructor/Section: _____

*"I pledge by my honor that I will abide by the
NJIT Academic Integrity Code."*

_____ (Signature)

Problem(s)	Score	Total

Relevant Formulas for this Exam

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$
$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
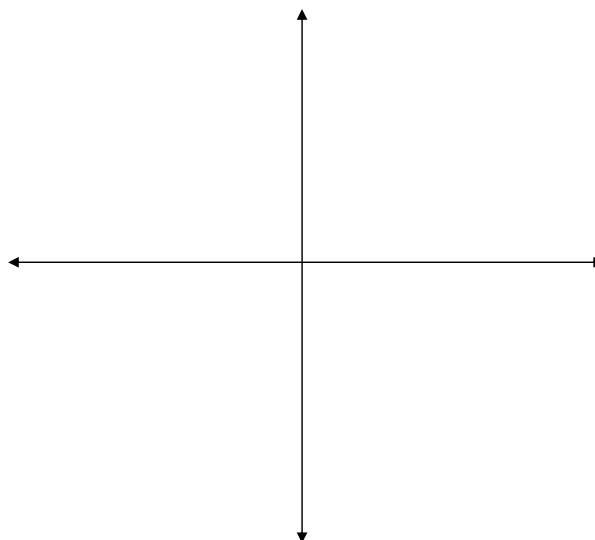$$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
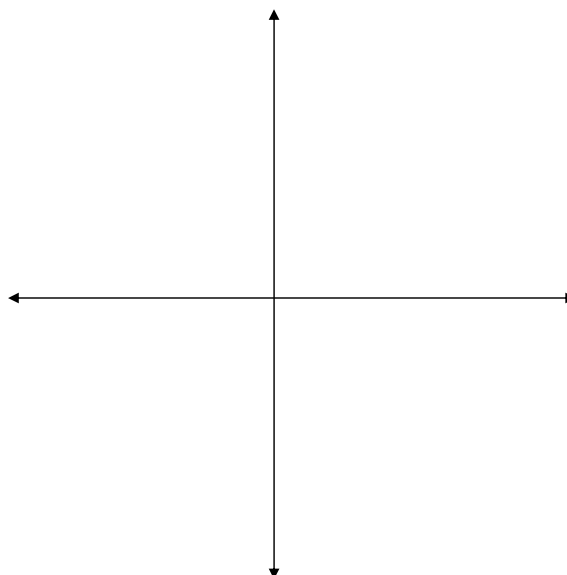
Rationalize any and all denominators, if necessary.

1. a) **(8 pts)** Graph <u>two periods</u> of the trigonometric function below. Clearly show and label any asymptotes and zeros.
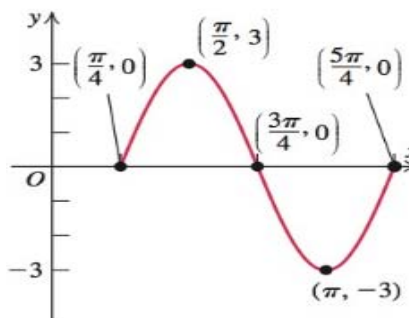
$$y = 2\tan\left(x - \frac{\pi}{2}\right)$$

b) **(8 pts)** Graph one period of the function. Clearly show and label all key quarter period points.

$$y = -3\sin\left[\frac{\pi}{2}(x + 1)\right]$$

c) **(4 pts)** The graph of a sinusoidal function is given below. Write an equation for a sine wave of the general form: $y = a\sin b(x - c)$ *(where **a, b** and **c** are unknown constants)* that when graphed will result in the given graph below.

2. **(8 pts)** Use an appropriate trigonometric identity and find the exact value of the following. ***Draw a quadrant diagram, if necessary.***

a) $\cos\left(\dfrac{\pi}{8}\right)$

b) $\sin(105°)$

3. **(15 pts)** Evaluate the following. ***Draw a quadrant diagram, if necessary.***

a) $\sin\left(\frac{32\pi}{3}\right)$

b) $\tan(-135°)$

c) $\sin^{-1}\left(-\frac{1}{2}\right)$

d) $\cos^{-1}\left[\cos\left(\frac{\pi}{3}\right)\right]$

e) $\sin\left[\frac{\pi}{2} + \tan^{-1}(1)\right]$

4. **(16 pts)** If $\cos \alpha = \frac{1}{2}$ with α in Quad. IV, and $\tan \beta = -\frac{5}{12}$ with β in Quad. II, find the following. *Draw a quadrant diagram, if necessary.*

a) $\cos(\alpha + \beta)$

b) $\sin(\alpha - \beta)$

c) $\sin(2\beta)$

d) $\cos(2\alpha)$

5. A Ferris wheel is built such that the height h (in feet) above ground of a seat on the wheel at time t (in seconds) can be modeled by: $h(t) = 53 + 50 \sin\left(\frac{\pi}{10}t - \frac{\pi}{2}\right)$,

    a) **(2 pts)** Find the period of the model. What does the period tell you about the ride?

    b) **(4 pts)** Find the maximum and minimum height h (in feet) above ground of a seat on the wheel during the ride and a possible time when each may occur.

    c) ***Extra Credit:*** **(2 pts)**
        What is the diameter of the wheel?

6. Use the fundamental trigonometric identities and the appropriate algebraic operations to perform the following:

a) **(4 pts)** Verify the identity: $\dfrac{\cos x}{1 - \sin x} = \tan x + \sec x$.

b) **(4 pts)** Simplify the trigonometric expression completely: $1 + \cos(2x) + 2\sin^2(x)$.

c) **(5 pts)** Use the double angle identities to find a triple angle identity for $\cos 3x$ in terms of $\cos x$ only.

7. **(12 pts)** Given that $\cos\theta = \dfrac{a}{b}$, with angle θ in quadrant I. Assume 'a' and 'b' are positive constants. Find the value of the following trigonometric expression in terms of 'a' and 'b'. ***Draw a quadrant diagram, if necessary.***

a) $1 - \sin^2\theta$

b) $3\cos^2(\theta) + 2\sin^2(\theta)$

c) $\sin\left(\dfrac{\pi}{2} - \theta\right)$

d) $(\tan^2\theta + 1)^3$

8. **(10 pts)** A wheel has a radius of 3 inches. A point on the rim of the wheel has the initial coordinates of $\left(-1, 2\sqrt{2}\right)$ relative to the center of the wheel. Find the coordinates of this point relative to the center of the wheel after the wheel rolls 23π inches to the left. ***Draw a quadrant diagram, if necessary.***