Math 110 Common Exam #1

September 27, 2017

Time: 1 hour and 25 minutes

Instructions: Show all work for full credit. No outside materials or calculators allowed.

Extra Space: Use the backs of each sheet for extra space. Clearly label when doing so.

Name: _____

ID #: _____

Instructor/Section: _____

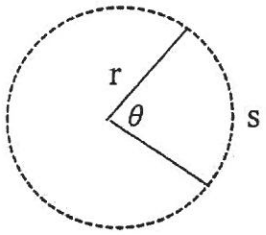
"I pledge by my honor that I have abided by the NJIT Academic Integrity Code."

_____ (Signature)

Problem(s)	Score	Total

Relevant Formulas for this Exam:

Circular motion and equations relating to a sector of a circle, radius r (as shown to the right).



$s = r\theta$

$v = r\omega$

$A = \frac{1}{2}r^2\theta$

$P = P_0 e^{kt}$

1. **(16 pts)** Find the exact value of the following expression.

a) $2\ln\sqrt{e} + 2^{\log_2 4} + \log_2[16]$

$2\ln e^{\frac{1}{2}} + 2^{\log_2 4} + \log_2 2^4$

$2(\frac{1}{2}) + 4 + 4$

$1 + 4 + 4$

9

b) $\log_3 135 - \log_3 45$

$\log_3\left(\frac{135}{45}\right) = \log_3 3$

$= 1$

c) $\log_2[e^{\ln 8}]$

$\log_2[8]$

$\log_2 2^3$

3

d) $2[\cos(45°)]^2 + 2[\sin(45°)]^2 - 2\tan(45°)$

$2\cos^2 45° + 2\sin^2 45° - 2\tan 45°$

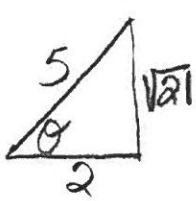
$2(\cos^2 45° + \sin^2 45°) - 2\tan 45°$

$2(1) - 2(1)$

$2 - 2$

0

2. **(6 pts)** Given that $\cos\theta = \frac{2}{5}$, where θ is an acute angle, find the exact values of the 5 other trigonometric functions. ***(Rationalize any and all denominators).***



$y = \sqrt{5^2 - 2^2}$

$= \sqrt{25 - 4}$

$= \sqrt{21}$

$\cos\theta = \frac{2}{5}$

$\sin\theta = \frac{\sqrt{21}}{5}$

$\tan\theta = \frac{\sqrt{21}}{2}$

$\sec\theta = \frac{5}{2}$

$\csc\theta = \frac{5}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}}$

$= \frac{5\sqrt{21}}{21}$

$\cot\theta = \frac{2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}}$

$= \frac{2\sqrt{21}}{21}$

3. a) **(4 pts)** Write the expression in condensed (compressed) form with a coefficient of 1. (***Assume all variables represent positive numbers***)

$\frac{1}{3}\log(x+2)^3 + \frac{1}{2}[\log(x^4) - \log(x^2 - x - 6)^2]$

$\frac{1}{3}(3)\log(x+2) + \frac{1}{2}\left[\log \frac{x^4}{(x^2-x-6)^2}\right]$

$\log(x+2) + \log\left[\frac{x^4}{(x^2-x-6)^2}\right]^{\frac{1}{2}}$

$\log(x+2) + \log\left[\frac{x^2}{x^2-x-6}\right]$

$\log\left[\frac{(x+2)x^2}{x^2-x-6}\right] = \log\left[\frac{(x+2)x^2}{(x-3)(x+2)}\right] = \log\left[\frac{x^2}{x-3}\right]$

b) **(4 pts)** Write the expression in expanded form. (***Assume all variables represent positive numbers***)

$\log\sqrt{100x\sqrt{y}} = \log\left[100x\sqrt{y}\right]^{\frac{1}{2}}$

$= \frac{1}{2}\log\left[100x\, y^{\frac{1}{2}}\right]$

$= \frac{1}{2}\left[\log 100 + \log x + \log y^{\frac{1}{2}}\right]$

$= \frac{1}{2}\left[\log 10^2 + \log x + \log y^{\frac{1}{2}}\right]$

$= \frac{1}{2}\left[2 + \log x + \frac{1}{2}\log y\right]$

$= \frac{1}{2}(2) + \frac{1}{2}\log x + \frac{1}{2}\cdot\frac{1}{2}\log y = 1 + \frac{1}{2}\log x + \frac{1}{4}\log y$

4. (10 pts) Graph the function $y = -2^{x+2} - 1$, on the set of axes below by making a table of values or using transformation. Be sure to label the asymptote on the graph, if any exists.

$y = 2^x$

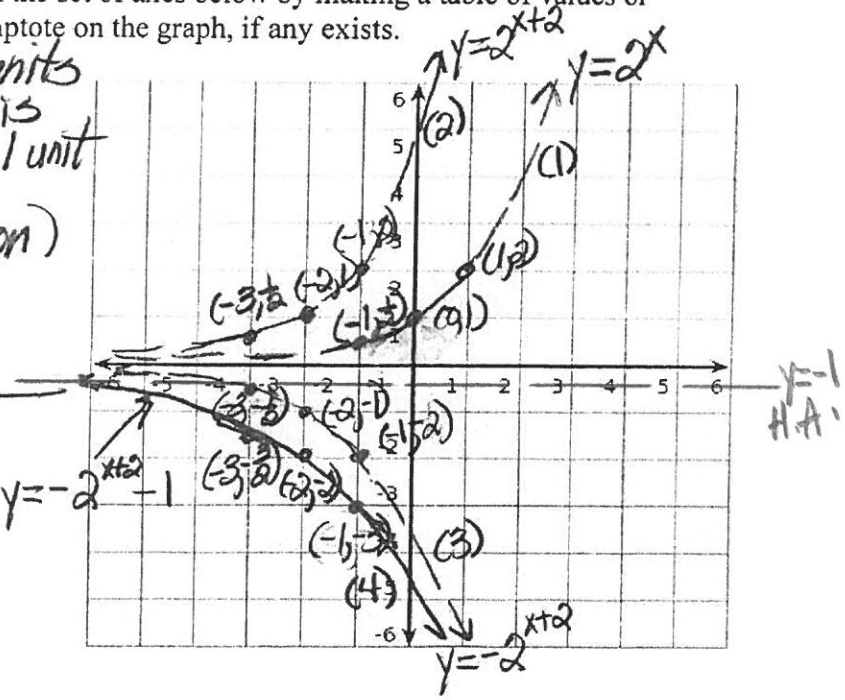
x	y
0	1
1	2
-1	$\frac{1}{2}$

shifts left by 2 units
Reflect about x-axis
vertical shift down 1 unit

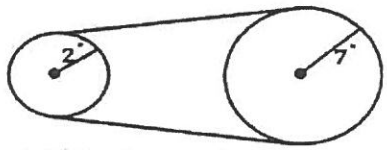
(Using Transformation)

Using Table of Values

x	y
-3	$-\frac{3}{2}$
-2	-2
-1	-3



5. Given the 2 in. radius wheel and 7 in. radius wheel pulley system as shown below, find the following. ***(You Can't Use Ratios).***



a) **(5 pts)** If the 2 in. radius wheel turns through an angle of 50°, what angle (expressed in degrees) does the 7 in. radius wheel turns through.

2in. wheel

$r_2 = 2$ in.

$\theta_2 = \cancel{50}^{5}\,\text{deg}\left(\frac{\pi\,\text{rad}}{\cancel{180}_{18}\,\text{deg}}\right)$

$= \frac{5\pi}{18}$ rad

$s_2 = r\theta$

$= \cancel{2}_{1}\left(\frac{5\pi}{\cancel{18}_{9}}\right)$

$= \frac{5\pi}{9}$ in.

7in. wheel

$r = 7$ in.

$s_7 = s_2 = \frac{5\pi}{9}$ in.

$\theta_7 = \frac{s_7}{r_7} = \frac{\frac{5\pi}{9}}{7} = \frac{5\pi}{63}$ rad

$\theta_{deg} = \frac{5\pi}{\cancel{63}_{7}}\,\text{rad}\left(\frac{\cancel{180}^{20}\,\text{deg}}{\cancel{\pi}\,\text{rad}}\right)$

$= \frac{100}{7}$ deg

b) **(5 pts)** If the 2 in. radius wheel is spinning at a rate of 6 rpms, how many rpms is the 7 in. radius wheel making?

2 in. wheel

$r_2 = 2$ in.

$\omega_2 = \frac{6\,\text{rev}}{\text{min}}\left(\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right) = \frac{12\pi\,\text{rad}}{\text{min}}$

$v_2 = r_2\omega_2$

$= 2(12\pi) = 24\pi$ in/min

7in. wheel

$r_7 = 7$ in.

$v_7 = v_2 = 24\pi$ in/min

$\omega_7 = \frac{v_7}{r_7} = \frac{24\pi}{7}$ rad/min

$\omega_{rpm} = \frac{\cancel{24\pi}^{12}}{7}\,\frac{\cancel{\text{rad}}}{\text{min}}\left(\frac{1\,\text{rev}}{\cancel{2\pi\,\text{rad}}_{1}}\right)$

$= \frac{12}{7}$ rpm

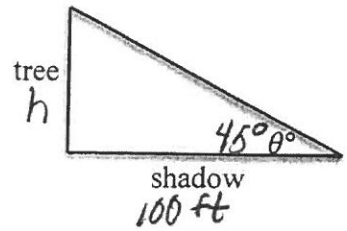
6. a) **(5 pts)** Find the height of a tree that casts a 100-feet shadow on the ground if θ (the angle of elevation) to the sun from the ground is 45° as shown in the diagram below

$\tan 45^\circ = \frac{h}{100}$

$h = 100 \tan 45^\circ$

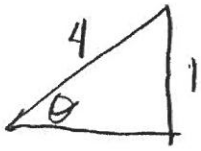
$= 100(1)$

$= 100 \text{ ft}$



$\tan 45^\circ = 1$

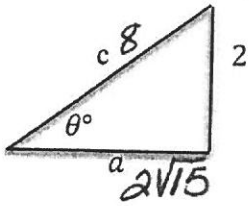
b) **(5 pts)** Given the right triangle as labeled below, if $\sin\theta = \frac{1}{4}$, find the lengths of side 'a' and 'c'. ***Simplify your answers as much as possible.***



Using similar triangles,

since $\sin\theta = \frac{1}{4} = \frac{2}{c}$,

then $c = 8$



Therefore, $a^2 = 8^2 - 2^2$

$= 64 - 4$

$= 60$

$a = \pm 2\sqrt{15}$

$a = 2\sqrt{15}$

7. **(20 pts)** Solve the following equation for all real solutions. Make sure all answers are in the domain of the original problem.

a) $\log_6(x+2) + \log_6(x-3) = 1$

$\log_6[(x+2)(x-3)] = 1$

$(x+2)(x-3) = 6^1$

$x^2 - x - 6 = 6$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4$ $\quad \cancel{x = -3}$

don't work in arguments of original equations

$\{4\}$

b) $3xe^x + x^2e^x = 0$

$xe^x(3+x) = 0$

$x = 0$ $\quad \cancel{e^x = 0}$ $\quad 3 + x = 0$

$x = -3$

e^x can never be equal to 0

$\{0, -3\}$

c) $4(2^{6-2x}) - 5 = 27$

$4(2^{6-2x}) = 32$

$2^{6-2x} = 8 = 2^3$

$6 - 2x = 3$

$-2x = -3$

$x = \frac{3}{2}$

$\{\frac{3}{2}\}$

d) $\log(x^2+1) = \log(x-2) + \log(x+3)$

$\log(x^2+1) = \log[(x-2)(x+3)]$

using one-to-one Property:

$x^2 + 1 = (x-2)(x+3)$

$x^2 + 1 = x^2 + x - 6$

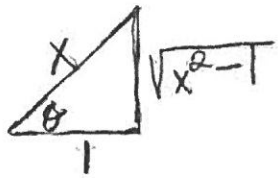
$\cancel{x^2} + 1 - \cancel{x^2} - x + 6 = 0$

$7 - x = 0$

$x = 7$

$\{7\}$

8. a) **(5 pts)** Suppose that $\cos\theta = \frac{1}{x}$ where 'x' is a nonzero constant. Find the values of the other 5 trigonometric functions in terms of 'x'. ***(You do not need to rationalize the denominator)***



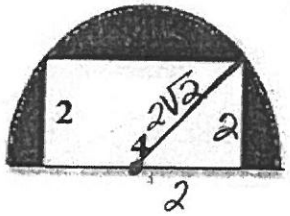
$\cos\theta = \frac{1}{x}$ $\quad$ $\sec\theta = x$

$\sin\theta = \frac{\sqrt{x^2-1}}{x}$ $\quad$ $\csc\theta = \frac{x}{\sqrt{x^2-1}}$

$\tan\theta = \frac{\sqrt{x^2-1}}{1}$ $\quad$ $\cot\theta = \frac{1}{\sqrt{x^2-1}}$

$= \sqrt{x^2-1}$

b) **(5 pts)** Given the rectangle (4 feet by 2 feet) inscribed in the semicircle as shown below, find the area of the shaded region of the semicircle.



Area of Rectangle $= 4 \cdot 2 = 8\ ft^2$

Radius of semicircle $= 2\sqrt{2}$

Area of semicircle $= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi(2\sqrt{2})^2$

$= \frac{1}{2}\pi(8)$

$= 4\pi\ ft^2$

Area of shaded region of semicircle = Area of semicircle − Area of rectangle

$= (4\pi - 8)\ ft^2$

$= 4(\pi - 2)\ ft^2$

9. a) **(4 pts)** Given that $f(x) = 5^x + 5^{-x}$ and $g(x) = 5^x - 5^{-x}$, evaluate the following expression below. ***Fully simplify your answer.***

$[f(x)]^2 + [g(x)]^2$

$$[f(x)]^2 = [5^x + 5^{-x}]^2 = 5^{2x} + 2\cdot 5^x \cdot 5^{-x} + 5^{-2x} = 5^{2x} + 2\cdot 5^0 + 5^{-2x}$$
$$= 5^{2x} + 2 + 5^{-2x}$$

$$[g(x)]^2 = [5^x - 5^{-x}]^2 = 5^{2x} - 2\cdot 5^x \cdot 5^{-x} + 5^{-2x} = 5^{2x} - 2\cdot 5^0 + 5^{-2x}$$
$$= 5^{2x} - 2 + 5^{-2x}$$

Therefore, $[f(x)]^2 + [g(x)]^2 = (5^{2x} + 2 + 5^{-2x}) + (5^{2x} - 2 + 5^{-2x})$

$$= 5^{2x} \cancel{+2} + 5^{-2x} + 5^{2x} \cancel{-2} + 5^{-2x}$$
$$= 2\cdot 5^{2x} + 2\cdot 5^{-2x} = 2[5^{2x} + 5^{-2x}]$$

b) **(3 pts)** Simplify completely ***(give your answer with positive exponents only)***

$$\left[\frac{(-x^2y)^3y^{-4}}{(xy)^5}\right]^{-2} = \left[\frac{-x^6y^3y^{-4}}{x^5y^5}\right]^{-2}$$
$$= \left[\frac{-x^6y^{-1}}{x^5y^5}\right]^{-2}$$
$$= \left[\frac{-x}{y^6}\right]^{-2} = \left[\frac{y^6}{-x}\right]^2 = \frac{y^{12}}{x^2}$$

c) **(3 pts)** Simplify completely.

$$\frac{\sqrt{16-16x^2}}{4-4x} = \frac{\sqrt{16(1-x^2)}}{4(1-x)}$$
$$= \frac{4\sqrt{1-x^2}}{4(1-x)}$$
$$= \frac{\sqrt{1-x^2}}{1-x}$$